

AL.2.2003-27

University of Alberta Library



0 1620 3409708 7

Module 4

Applied Mathematics 20

LINEAR PROGRAMMING



Learning
Technologies
Branch

Alberta
LEARNING



Digitized by the Internet Archive
in 2016 with funding from
University of Alberta Libraries

Module

4



Applied Mathematics 20

LINEAR PROGRAMMING

Applied Mathematics 20
Module 4: Linear Programming
Student Module Booklet
Learning Technologies Branch
ISBN 0-7741-1990-X

Cover: **(Collage)** PhotoDisc, Inc.
Title page: PhotoDisc, Inc.

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	



You may find the following Internet sites useful:

- Alberta Learning, <http://www.learning.gov.ab.ca>
- Learning Technologies Branch, <http://www.learning.gov.ab.ca/ltb>
- Learning Resources Centre, <http://www.lrc.learning.gov.ab.ca>

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

ALL RIGHTS RESERVED

Copyright © 2001, the Crown in Right of Alberta, as represented by the Minister of Learning, Alberta Learning, 11160 Jasper Avenue, Edmonton, Alberta T5K 0L2. All rights reserved. Additional copies may be obtained from the Learning Resources Centre.

No part of this courseware may be reproduced in any form, including photocopying (unless otherwise indicated), without the written permission of Alberta Learning.

Every effort has been made both to provide proper acknowledgement of the original source and to comply with copyright law. If cases are identified where this effort has been unsuccessful, please notify Alberta Learning so that appropriate corrective action can be taken.

IT IS STRICTLY PROHIBITED TO COPY ANY PART OF THESE MATERIALS UNDER THE TERMS OF A LICENCE FROM A COLLECTIVE OR A LICENSING BODY.

Welcome

Applied Mathematics 20

Module 1
Graphs

Module 2
Non-Linear Functions

Module 3
Linear Systems

Module 4
Linear Programming

Module 5
Finance

Module 6
Circles

Module 7
Measurement

*Welcome
to Module 4.
We hope you'll
enjoy your
study of
Linear Programming.*



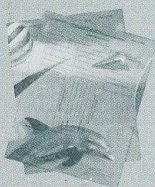
Applied Mathematics 20 contains seven modules and a final test. Work through the modules in the order given, since several concepts build on each other as you progress through the course.

Contents

Introduction to Applied Mathematics 20	6
Module Overview	
Assessment	13
Module Project: Trail Mix	
Beginning the Project	14
Activity 1:	
The Graph of a Linear Inequality	15
Activity 2:	
Graphing a Linear Inequality in Two Variables	21
Activity 3:	
The Solution of a System of Linear Inequalities	26
Activity 4:	
Modelling a Problem Situation	29
Activity 5:	
Optimization Problems	32
Follow-up Activities	
Extra Help	37
Enrichment	39

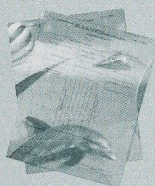
Module Project: Trail Mix

Completing the Project----- 43



Submit the Module Project.

Module Summary----- 46



Submit the Module Assignment.

Appendix

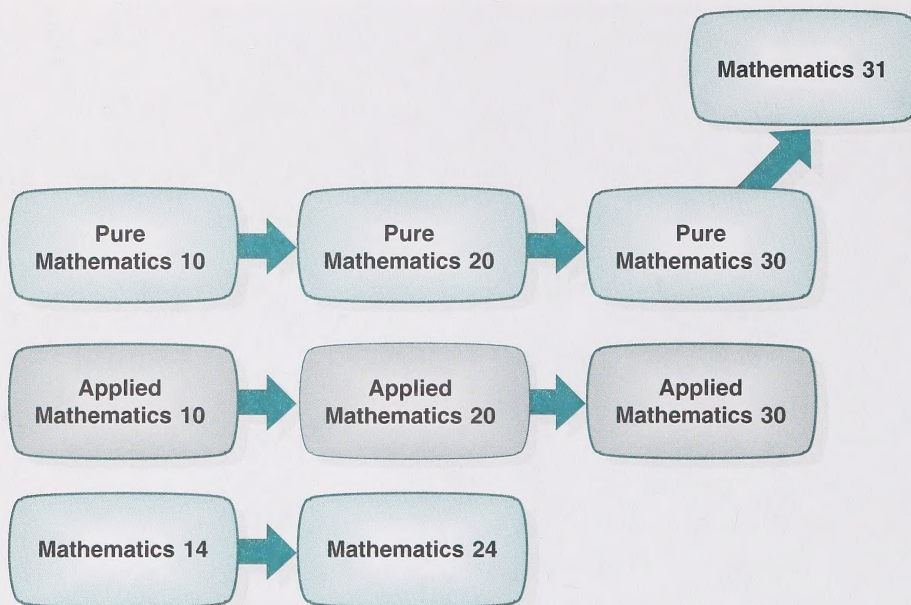
Glossary ----- 48

Suggested Answers ----- 48

Image Credits ----- 102

Introduction to Applied Mathematics 20

Applied Mathematics 20 is the second course in the Applied Mathematics 10–20–30 program of studies. Another program of studies is Pure Mathematics 10–20–30; students who complete Pure Mathematics 30 often choose to take Mathematics 31. A third program of studies is Mathematics 14–24.



Each mathematics program is designed for students with different mathematical strengths and interests.

- Pure Mathematics 10–20–30 is intended for students who are strong in algebra and mathematical theory.
- Applied Mathematics 10–20–30 is better suited to students who prefer to solve problems using numerical reasoning or geometry.
- Mathematics 14–24 is a general mathematics program for high school students who have experienced difficulties in previous mathematics courses.

Each sequence of courses is designed for students with different career plans. For example, Pure Mathematics 30 is a prerequisite for admission to many university programs. Many colleges and technical institutes, however, will admit students who have successfully completed Applied Mathematics 30.

You may find it helpful to read “Questions and Answers About Senior High School Mathematics” and “The New Senior High School Mathematics Program and Post-Secondary Studies.” These documents can be found at the following Internet site:

http://www.learning.gov.ab.ca/k_12/curriculum/bySubject/math

Before enrolling in Applied Mathematics 20, it is recommended that you talk with a school counsellor about your career plans.



TRANSFERRING FROM THE APPLIED PROGRAM

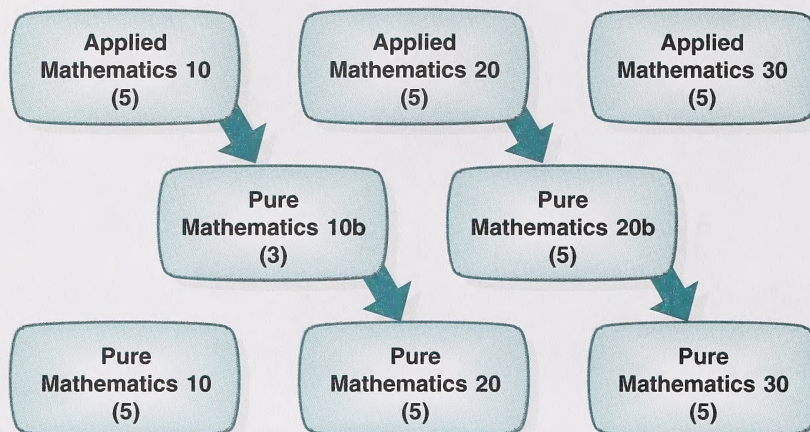
You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are independent.

The following table shows some common and independent topics.

Applied Topics	Common Topics	Pure Topics
• linear programming • data tables and trends • design and layout • metric and imperial measure • data presentation • vectors and matrices • periodic, fractal, and recursive patterns • financial decision making • costing and design problems	• spreadsheets • line segments and linear graphs • scaling • triangles • surveys • financial mathematics • quadratic functions • circle geometry • the bell curve	• irrational numbers • exponents • polynomial and rational expressions • mathematical expectation • growth patterns • linear and non-linear systems • operations on functions • mathematical reasoning • exponential and logarithmic functions • conics • combinations • trigonometric functions

If you want to transfer from the Applied Mathematics 10–20–30 sequence to the Pure Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics.

If you decide to transfer to Pure Mathematics 20 after successfully completing Applied Mathematics 10, you may have to take a three-credit course called Pure Mathematics 10b. If you decide to transfer to Pure Mathematics 30 after successfully completing Applied Mathematics 20, you may have to take a five-credit course called Pure Mathematics 20b. The two bridging courses are shown in the following diagram.

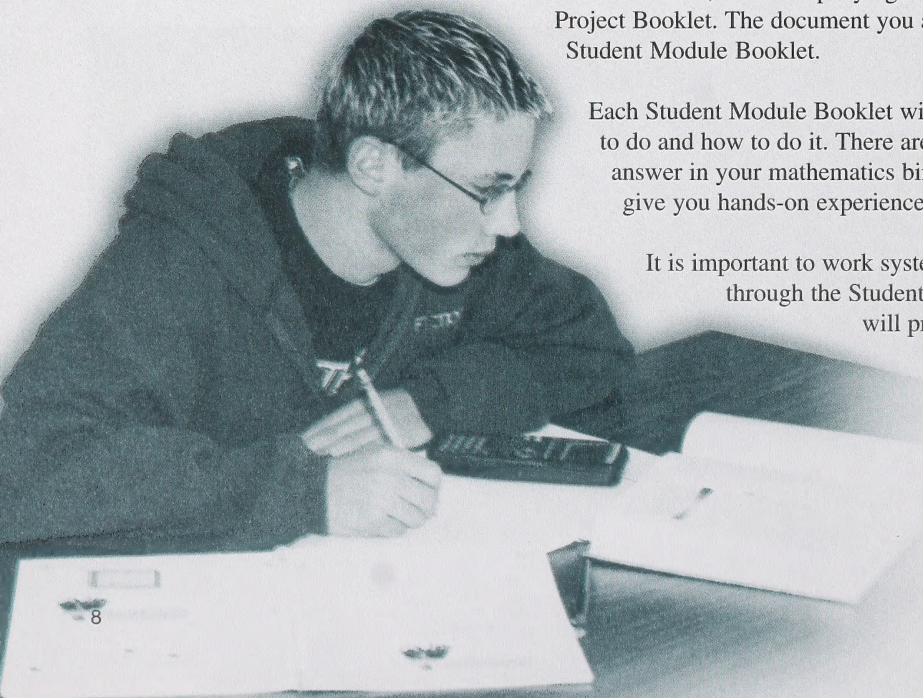


STRATEGIES FOR COMPLETING APPLIED MATHEMATICS 20

For each module in Applied Mathematics 20, there is a Student Module Booklet, an accompanying Assignment Booklet, and a Project Booklet. The document you are presently reading is called a Student Module Booklet.

Each Student Module Booklet will show you, step by step, what to do and how to do it. There are readings, questions for you to answer in your mathematics binder, and applications that will give you hands-on experience.

It is important to work systematically and carefully through the Student Module Booklets. This work will prepare you for the assignments, projects, and final test.



Following are some suggestions for organizing your mathematics binder:

- Keep a section of your binder to record your responses to the questions in the Student Module Booklet. Also store your marked assignments here.
- Keep a section of your binder for work in progress on your projects. Keep your research notes, plans, rough drafts, and so on.
- Keep a section of your binder to record new skills and concepts, as well as important results and formulas. Get in the habit of describing new skills and concepts in your own words.

Record useful ways to help you remember what a concept means. Make charts and diagrams to help you connect mathematical ideas.

- Keep a section of your binder to record mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you grasped a difficult concept (an “aha!” experience), or when you used a calculator or spreadsheet in a new way.

Mathematical Processes

Throughout this course, you will be expected to perform the following mathematical processes:

- Connect mathematical ideas to everyday experiences and to concepts in other disciplines.
- Develop and use problem-solving strategies.
- Reason and justify your answers.
- Communicate mathematical ideas.
- Select and use appropriate technologies to solve problems.
- Develop and use estimation and mental-math strategies.
- Use visualization to assist in processing information, making connections, and solving problems.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Applied Mathematics 20 to be your study partner. You will find that having a friend to discuss mathematical ideas with will make your studying more enjoyable.



Resources You Will Need

In addition to the distance learning materials for Applied Mathematics 20, you will need the following resources:

- the *Addison-Wesley Applied Mathematics 11 Source Book*, Western Canadian Edition, published by Addison Wesley Longman Ltd. (2000)
- a binder, lined loose-leaf paper, graph paper, dividers, pencils, eraser
- metric and imperial measuring devices, such as a ruler, yardstick, metre-stick, and tape measure
- a mathematical instrument set (compass, protractor, and triangles)
- a computer with a spreadsheet program

Note: Two popular spreadsheet programs are *ClarisWorks™* and Microsoft® *Excel*.

- a graphing calculator

Note: Where it is applicable, the examples in this course and the textbook show the TI-83 calculator; however, all of the graphing calculators in the following chart are approved for use on tests.

Texas Instruments	Sharp	Casio	Hewlett-Packard
TI-82** TI-83 TI-83 Plus TI-86 TI-89 TI-92* TI-92 Plus	EL-9600C EL-9600* EL-9200* EL-9300*	Algebra FX 2.0 CFX-9850 GA-Plus* CFX-9850 G* CFX-9800 G* FX-9700 series*	HP 39g

*no longer commercially available

**The TI-82 calculator will remain on the approved list for the 2000–2001 and 2001–2002 school years and will then be deleted from the approved list.

If you intend to use the TI-83 or TI-83 Plus graphing calculator, it is recommended that you obtain the video program *The TI-83 Graphing Calculator Video Tutor*.

Many of the resources that you will need may be purchased locally or from the Learning Resources Centre (LRC). Following is the LRC website:

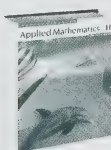
<http://www.lrc.learning.gov.ab.ca>

You may wish to discuss the availability of resources with your teacher, as your school division may have a loan policy.

Visual Cues

You will find many visual cues in this course. Colour is used to highlight terms that are defined in the Glossary of the Appendix of each Student Module Booklet. You will also find several icons in the margins. Read the following explanations to discover what the various icons prompt you to do.

- Refer to the textbook.



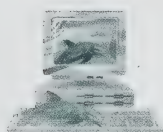
- Refer to the Applied Mathematics 20 CD.



- Use mathematical instruments or measuring tools.



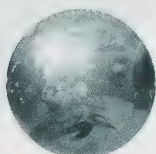
- Work with a computer.



- Complete the module project or assignment.



- Explore the Internet.



Remember: Any Internet website address given in this module is subject to change.

MODULE OVERVIEW





Do you go hiking or camping? Do you take a mixture of nuts, raisins, and chocolate with you as a snack? Do you know how the ingredients for such a mixture are determined so that at least a certain amount of nutrition is provided and that the cost is minimal?

Linear programming involves modelling problem situations with systems of linear inequalities and using these systems to find possible solutions as well as the optimal solution. The optimal solution is the one that meets specific goals, such as minimizing costs, improving efficiency, and maximizing profits.

In this module you will graph inequalities in two variables on a coordinate plane. You will then solve linear and non-linear systems of inequalities, and you will use these systems to model problem situations and find feasible solutions.

Finally, you will determine the best solution: one that meets a specific goal.

ASSESSMENT

Accompanying this Student Module Booklet is a Project Booklet and an Assignment Booklet. Your grading in this module will be based upon the module project and the module assignment you submit for evaluation.

The mark distribution is as follows:

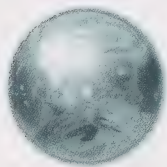
Module Project	40 marks
Module Assignment	60 marks

TOTAL 100 marks

Remember that Activities 1 to 5 in this Student Module Booklet will prepare you for completing the module project and the module assignment. You should work through these activities carefully and compare your answers with the suggested answers provided in the Appendix.

The Follow-up Activities provide extra help and enrichment. You may choose to do some or all the questions in the Follow-up Activities. Again, you should compare your answers with the suggested answers provided in the Appendix.

Beginning the Project



To begin your project, turn to page 168 of the textbook and read “Analyzing a Snack Food” and answer the questions posed. Store your responses in the project section of your mathematics binder.

Use the Internet, magazines, and the library to research nutrition and trail mixes. Early in your search for information, you may find it useful to visit Addison Wesley Longman Ltd.’s site as described on page 169 of the textbook. Select a topic under “Analyzing a Snack Food Activity.” There is no heading titled “Nutrition.”

In this project, you will focus on the system of linear inequalities that models the problem situation of designing a feasible trail mix. To model the problem situation, you will need a good understanding of systems of linear inequalities. As you work through Activities 1 to 5, you will develop the required understanding.

As you work through the activities, continue your research for this project. You will be given more direction on how to complete this project later in this module. In the meantime, continue exploring the project topic and keep notes, samples, and ideas in the project section of your mathematics binder. You may want to talk about this project with your study partner or with a family member. Remember, the work you submit must be your own.

ACTIVITY 1



The Graph of a Linear Inequality

To optimize is to make, design, or modify something so it serves its purpose in the best possible way.

The Concorde is a passenger aircraft that was optimized for the purpose of speed. In the design of this aircraft, maximizing speed was not the only consideration. Fuel consumption, passenger capacity, and flight stability would also be conditions that were likely imposed on the aircraft's design team.

Designing the Concorde was like solving an optimization problem. Optimization problems are problems in which you aim for a maximum or minimum value subject to certain conditions. For certain optimization problems, you use a **system of linear inequalities** to model the situation.

Turn to page 170 of the textbook and read the introductory paragraphs of Tutorial 4.1, "The Graph of a Linear Inequality."

Graphing inequalities on a number line requires a number of symbols to indicate the starting position and the direction of the solution. An open circle is used to show that the number is not included in the solution, and a closed circle is used to show that the number is part of the solution. A coloured arrow shows the direction of the solution.



Example

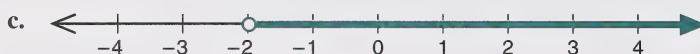
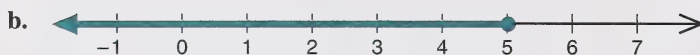
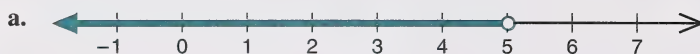
Graph the following.

a. $w < 5$

b. $w \leq 5$

c. $w > -2$

Solution



1. Turn to page 170 of the textbook and answer exercises 1, 2, 3.d., and 4.e. of “Practise Your Prior Skills.”



Recall that when a linear equation is expressed in the form $y = mx + b$, m represents the slope and b the y -intercept.

2. Turn to pages 170 and 171 of the textbook and answer exercises 5.a. and 6.a. of “Practise Your Prior Skills.”

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 48–50.

On a number line, an equation having only one variable corresponds to just one point, which is indicated by a dot. On a coordinate plane (or grid), such an equation corresponds to a line, either horizontal or vertical.

Example

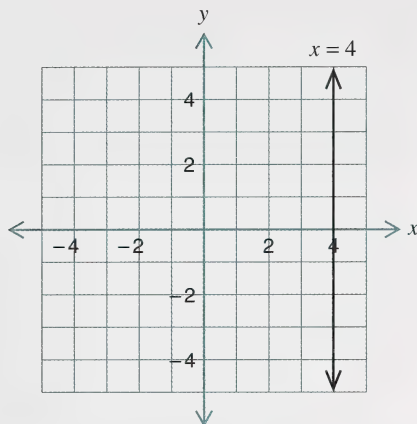
On a coordinate grid, graph the following.

a. $x = 4$

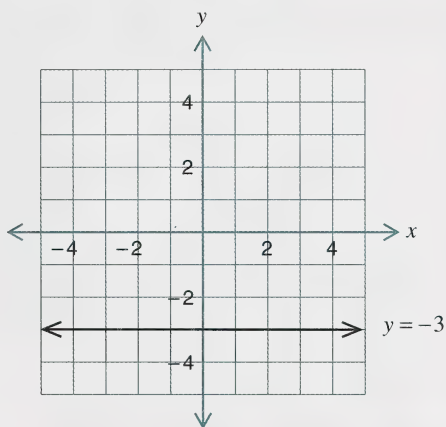
b. $y = -3$

Solution

a.



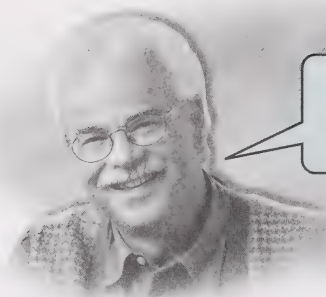
b.



Turn to page 171 of the textbook and read the paragraph following exercise 6 at the top of the page.

3. Answer exercises 1 to 10 of “Investigation 1: Graphing Half-Planes” on page 171 of the textbook.

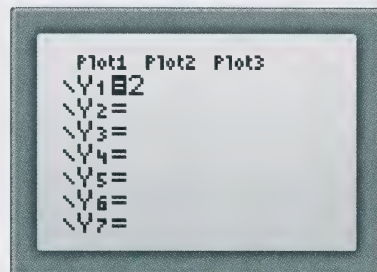
Compare your responses with the suggested answers in the Appendix, Activity 1, pages 51–53.



Do you know how to graph an inequality on your graphing calculator? If not, follow these steps on how to graph $y < 2$.

Step 1: Enter the corresponding equation,
 $y = 2$.

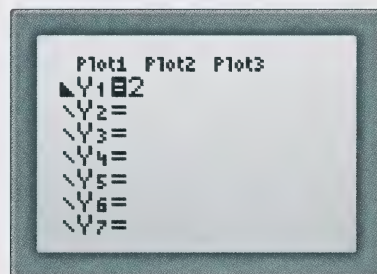
Y= 2



Step 2: Enter the **half-plane** that is to be shaded.

A half-plane is the part of the coordinate plane above or below a given line. A half-plane corresponds to a linear inequality.

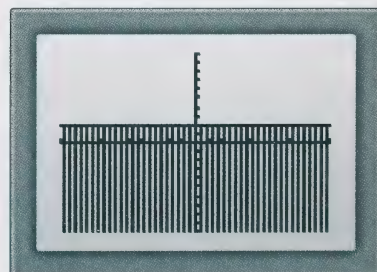
Move the cursor to the left of “Y₁” using the arrow keys; then press **ENTER** repeatedly to cycle through the graph-style icons, stopping at the Below icon. (Refer to the display.)



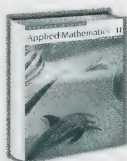
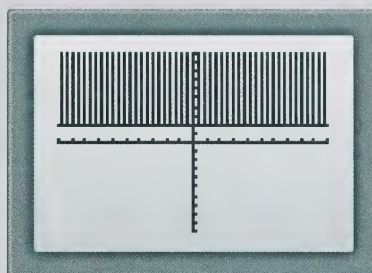
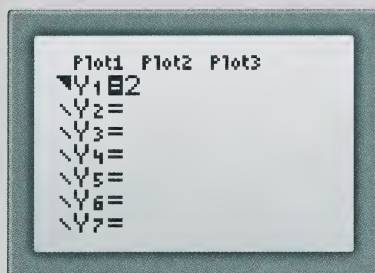
Step 3: Press **GRAPH** to display the inequality.

CAUTION

The graphing calculator does not show whether the boundary line is included or excluded.



Similarly, to graph $y > 2$, follow Steps 1 to 3 and choose the Above icon as the graph style rather than the Below icon.

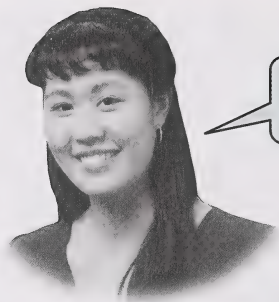


Turn to page 417 of the textbook and work through “Utility 21: Graphing Styles on the TI-83 Calculator” (or read the relevant section of your graphing calculator’s user’s guide) for more information about changing graph styles.

4. Answer exercises 1 and 2 of “Discussing the Ideas” on pages 172 and 173 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 1, page 53.





Now, you will graph inequalities in two variables.

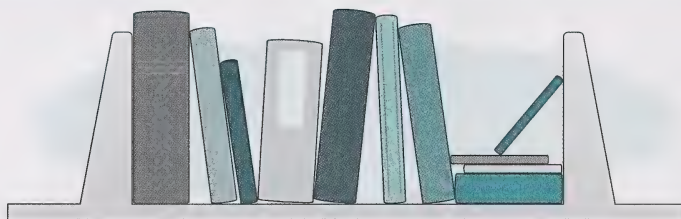


5. Turn to page 172 of the textbook and answer exercises 1 to 10 of “Investigation 2: Graphing an Inequality in Two Variables.”

Turn to page 172 of the textbook and read the paragraph preceding “Discussing the Ideas.”

6. Answer exercises 3 to 6 of “Discussing the Ideas” on page 173 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 54–57.



LOOKING BACK

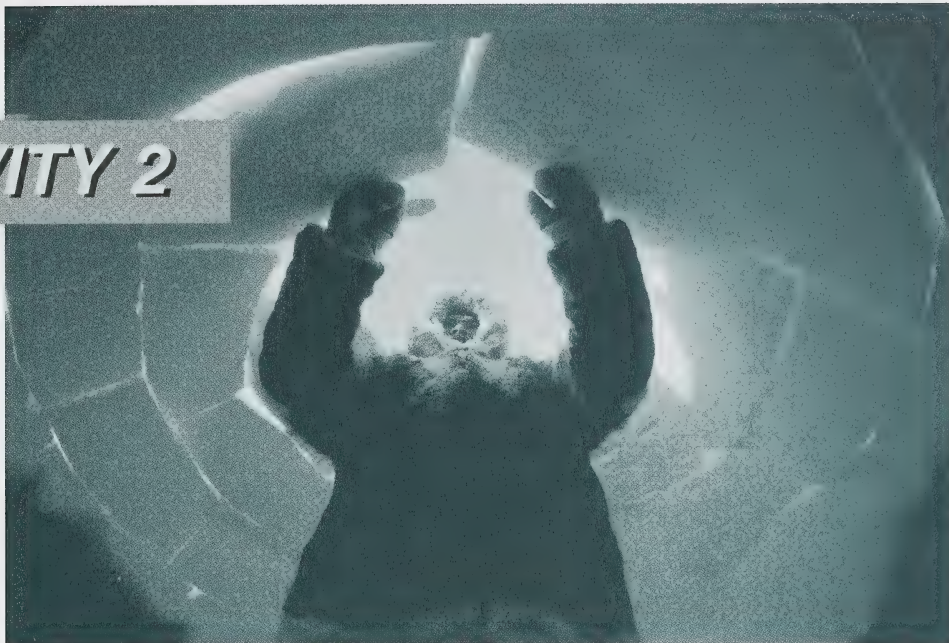


In this activity you investigated and developed methods of graphing linear inequalities using pencil and paper and using your graphing calculator. In your investigation, you examined how to use test points to help determine which half-plane to shade.

7. Turn to page 173 of the textbook and answer “Communicating the Ideas.”

Compare your response with the suggested answer in the Appendix, Activity 1, page 58.

ACTIVITY 2



Graphing a Linear Inequality in Two Variables

The Inuit in Northern Canada have built igloos for generations. To construct an igloo, you would most likely follow certain steps—steps based on the wisdom of Inuit builders. You would start with these steps.

Building an Igloo

- Cut out blocks of snow from a spot having hard, dry snow and that is suitable for the igloo site.
- Put the first row of blocks so they encircle the hole you created when cutting out the blocks of snow. Adjust the block edges so that the blocks fit tightly together.
- Build up the walls, shaping them so they lean inward.

To solve a linear inequality in two variables, you can also follow certain steps—steps based on the study of linear inequalities.



Turn to page 174 of the textbook and read the introductory paragraphs of Tutorial 4.2, “Graphing a Linear Inequality in Two Variables.” Pay particular attention to the procedure you can use to graph a linear inequality. Then work through “Example 1: Graph a Linear Inequality” on pages 174 to 176.

Example

Graph the inequality $4x - 2y \geq 6$. Check that you have shaded the correct half-plane.

Solution

First, solve the inequality for y .

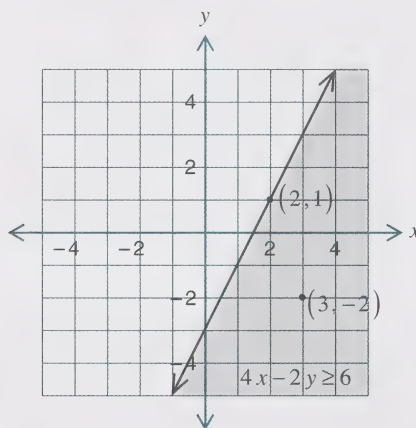
$$4x - 2y \geq 6$$

$$-2y \geq -4x + 6 \quad \leftarrow \text{Subtract } 4x \text{ from both sides.}$$

$$y \leq 2x - 3 \quad \leftarrow \text{Divide both sides by } -2, \text{ and switch the inequality sign.}$$

It is important to switch the inequality when dividing each side by a negative number.

In the inequality, the $\leq$ sign indicates that the boundary line is solid and that the half-plane below the boundary line is shaded. Therefore, graph the corresponding equation, $y = 2x - 3$, and shade the area below the line.



Check $(2, 1)$.

LS	RS
$4x - 2y$	6
$= 4(2) - 2(1)$	
$= 8 - 2$	
$= 6$	

LS $\geq$ RS

Check $(3, -2)$.

LS	RS
$4x - 2y$	6
$= 4(3) - 2(-2)$	
$= 12 + 4$	
$= 16$	

LS $\geq$ RS



If you are having trouble figuring out which side of the boundary line should be shaded, use a test point. A convenient test point to use is $(0, 0)$.

For example, use $(0, 0)$ to help you determine which half-plane in the preceding example should be shaded.

LS	RS
$4x - 2y$ $= 4(0) - 2(0)$ $= 0$	6
LS	RS

Because $(0, 0)$ does not satisfy the inequality, the half-plane on the other side of the boundary line should be shaded.



For more details and exercises involving using a test point, refer to the Extra Help section of this Student Module Booklet.

1. A student made the following notes of the steps involved when solving an inequality.

Apply a
test point.

Graph the line of the
corresponding
equation.

Express the inequality
in the form
 $y < mx + b$
or
 $y > mx + b$

Which step should you complete last? What is the purpose of this step?

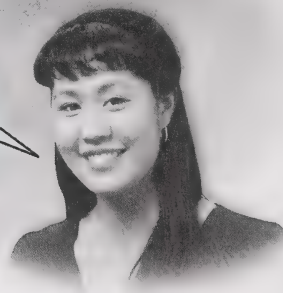


2. Answer exercises 1 and 4 of “Discussing the Ideas” on page 178 of the textbook.
3. Sketch the graph of the linear inequality $y < -\frac{3}{2}$. Use a test point to check that the correct half-plane has been shaded.
4. Answer exercises 2, 3.d., 3.f., 3.h., 4.b., 4.d., 5.b., and 6.b. of “Exercises: Checking Your Skills” on pages 178 and 179 of the textbook.

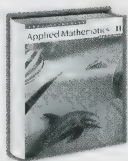
Compare your responses with the suggested answers in the Appendix, Activity 2, pages 58–62.



How can inequalities be used to model a situation algebraically?

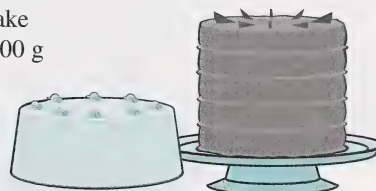


The next example will show you how.



Turn to pages 176 and 177 of the textbook and work through “Example 2: Solve an Inequality by Graphing.” Notice that the situation being modelled imposes restrictions on the domain and range of the inequality.

5. Answer exercises 2 and 3 of “Discussing the Ideas” on page 178 of the textbook.
6. Answer exercises 7 and 8 of “Exercises: Checking Your Skills” on page 179 of the textbook.
7. Sarah’s Bakery is making cakes. One type of cake requires 125 g of flour while another requires 100 g of flour. At most 2.50 kg of flour is available.



- a. Write the inequality that represents the problem.
- b. Use a graphing calculator to graph the inequality.
- c. Explain how the graph can be used to obtain feasible solutions to the problem.
- d. Describe two possible solutions.

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 62–64.

LOOKING BACK

In this activity, you extended your knowledge of graphing linear inequalities from the previous activity to include graphing linear inequalities involving two variables. You also used inequalities to model and solve situations.

8. A graphing calculator can be a great help when graphing inequalities. However, using pencil and paper has some advantages over using a graphing calculator. List some of these advantages.
9. How is the graph of a linear inequality different from the graph of a linear equation?

Compare your responses with the suggested answers in the Appendix, Activity 2, page 65.

ACTIVITY 3



The Solution of a System of Linear Inequalities

When roofing a house, it is important to have the shingles overlap. Having the shingles overlap ensures that water will run down and off the roof rather than leak through the roof.

Overlap is also important in graphing and solving systems of linear inequalities. A system of linear inequalities is a set of two or more linear inequalities considered at the same time. When graphing such a system, the half-planes corresponding to each of the inequalities may overlap in a region within the coordinate plane. This region represents the solution of the system.



Turn to page 181 of the textbook and read the introductory paragraphs of Tutorial 4.3, “The Solution of a System of Linear Inequalities.” Then work through “Example 1: Graph a System of Linear Inequalities” on pages 181 and 182.

1. Answer exercises 2, 3, 4.b., and 4.c. of “Exercises: Checking Your Skills” on pages 187 and 188 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 65–67.

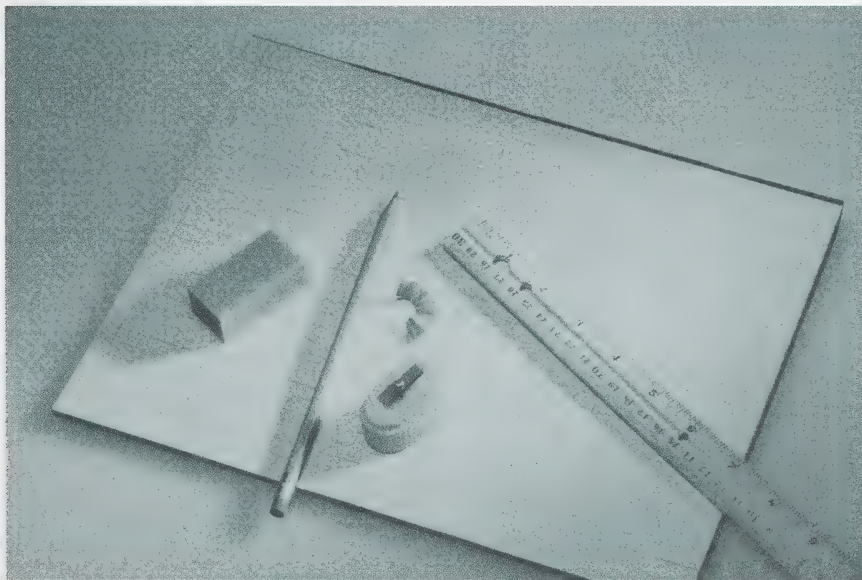
You have used your graphing calculator to graph linear inequalities. You can also use it to graph systems of linear inequalities.



Turn to pages 182 to 184 of the textbook and work through “Example 2: Graph a System of Inequalities Using Technology.” Notice that variables x and y are discrete variables. In this situation, they both represent the number of balls.

2. Answer exercise 1 of “Discussing the Ideas” on page 186 of the textbook.
3. Answer exercise 5 of “Exercises: Checking Your Skills” on page 188 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 67–68.



Another type of shading that can be used on a graphing calculator is **reverse shading**. It is used to make a solution more visible when three or more inequalities are graphed.



Turn to page 184 of the textbook and read the paragraph at the bottom of the page. Then work through “Example 3: Solve a System of Linear Inequalities Using Technology and Reverse Shading” on pages 185 and 186.

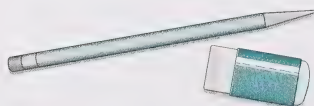
4. Answer exercises 4 and 5 of “Discussing the Ideas” on page 186 of the textbook.
5. Answer exercises 1, 6, and 7 of “Exercises: Checking Your Skills” on pages 187 and 188 of the textbook.

Hint for exercise 1: For graphs that use reverse shading, all points in the unshaded region satisfy each inequality of the system. You can choose a test point in the unshaded region to check your graph.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 68–70.

LOOKING BACK

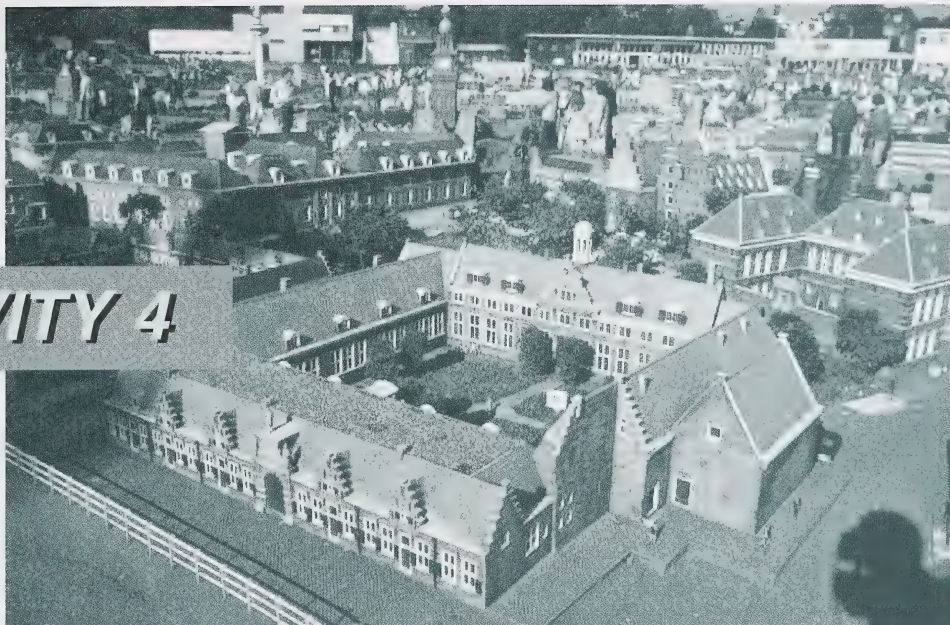
In this activity you graphed linear systems of inequalities and found solutions for these systems. You also modelled situations with linear systems of inequalities.



6. Turn to page 189 of the textbook and answer “Communicating the Ideas.”

Compare your responses with the suggested answers in the Appendix, Activity 3, page 71.

ACTIVITY 4



Modelling a Problem Situation

Madurodam is a miniature city built in the Dutch city of The Hague. Madurodam contains scale models of many famous buildings found throughout the Netherlands.

A model is a small copy or representation of an object or figure. In Madurodam, the models are physical (buildings, trees, etc.). In this activity you will make and use algebraic models. You will use systems of linear inequalities to model (represent) problem situations.



Turn to page 190 of the textbook and read the introductory paragraphs of Tutorial 4.4, "Modelling a Problem Situation."

1. Karamu works x hours each week at a hardware store and is paid \$7 per hour. Karamu also enjoys watching television; he watches y hours of television each week. Write an algebraic expression, equation, or inequality for each of the following.
 - a. the money Karamu earns each week
 - b. the time Karamu works each week if he works no more than 18 h
 - c. the time Karamu watches television if he watches a minimum of 5 h each week
 - d. the total time Karamu works and watches television if he never exceeds 25 h

Compare your responses with the suggested answers in the Appendix, Activity 4, page 71.

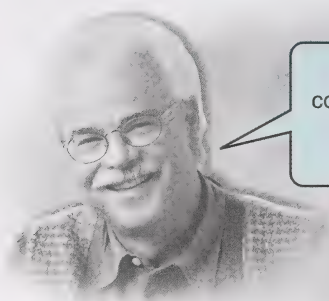
In exercise 1, there are constraints on variables x and y that are not stated: both x and y must be non-negative. But it can be inferred that both x and y are non-negative from the situation. These constraints are implied (or implicit) constraints. Watch for these in the next example.



Turn to pages 190 and 191 of the textbook and work through “Example 1: Developing an Exercise Program.”

2. Answer exercises 2 and 4 of “Exercises: Checking Your Skills” on pages 195 and 196 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 71–74.



In the next example, there is an implicit constraint that certain variables can only be whole numbers. This implicit constraint reduces the number of solutions.

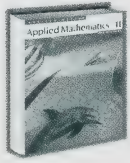


Turn to pages 192 and 193 of the textbook and work through “Example 2: Manufacturing Sportswear.”

3. Answer exercises 1 and 2 of “Discussing the Ideas” on page 195 of the textbook.
4. Answer exercise 5 of “Exercises: Checking Your Skills” on page 196 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 74–75.





The next example illustrates a situation where the system of inequalities needed to model a situation is not linear.

Turn to pages 194 and 195 of the textbook and work through “Example 3: Solve a Problem Using a Non-Linear System of Inequalities in Two Variables.”

5. Answer exercise 7 of “Exercises: Checking Your Skills” on page 197 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 75–76.



LOOKING BACK

In this activity you determined and solved systems of inequalities that modelled problem situations. You also recognized implicit constraints on variables in these problems.

6. When modelling a problem situation, what kind of algebraic statements do you use to represent the constraints of the situation?
7. Explain how implicit constraints of a problem situation differ from other constraints.

Compare your responses with the suggested answers in the Appendix, Activity 4, page 76.



ACTIVITY 5

Optimization Problems

Many problem situations have more than one solution. In that case, you would be wise to use the “best” solution. But what is the best solution? In a business context, the best solution is likely the one that yields the maximum profits.

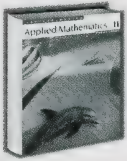
You have seen that when graphing a system of inequalities, the region of overlapping half-planes holds the solution set of the system. There may be many feasible points within the overlapping region—points that satisfy the system and are solutions of the given problem situation.

In optimization problems, you select the “best” solution of the ones available based on profits, efficiency, costs, or any other relevant criteria.

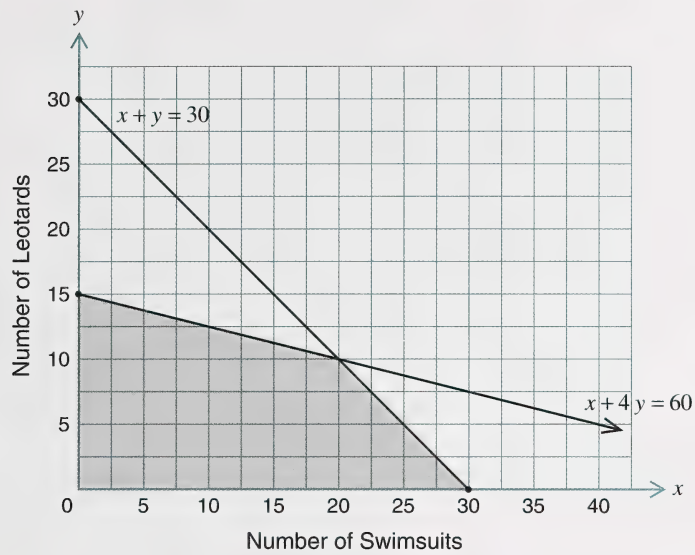
Turn to page 200 of the textbook and read the introductory paragraphs of Tutorial 4.5, “Optimization Problems.” You may want to review “Example 2: Manufacturing Sportswear” on pages 192 and 193 of the textbook.

The next investigation will explore maximizing profits using a two-dimensional coordinate plane and will lead you to the exact point of maximum profits within the solution region.

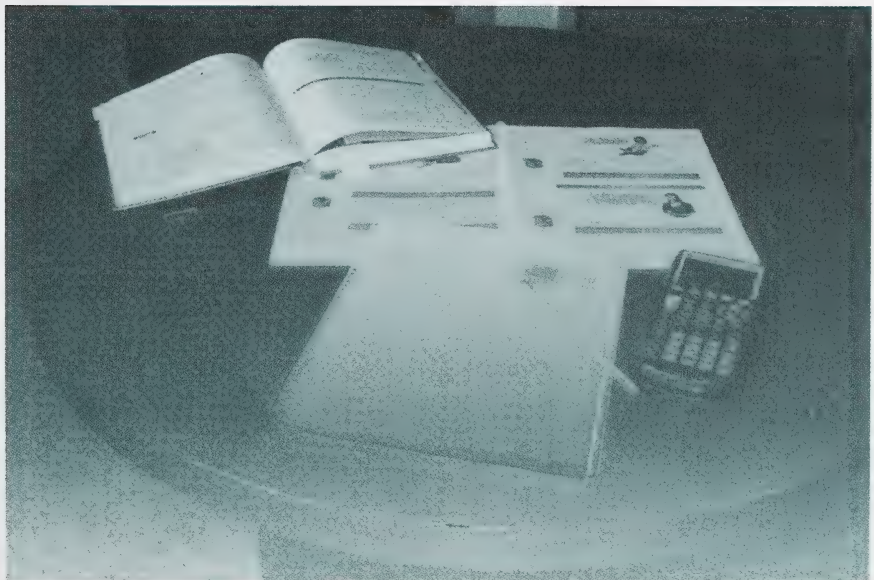




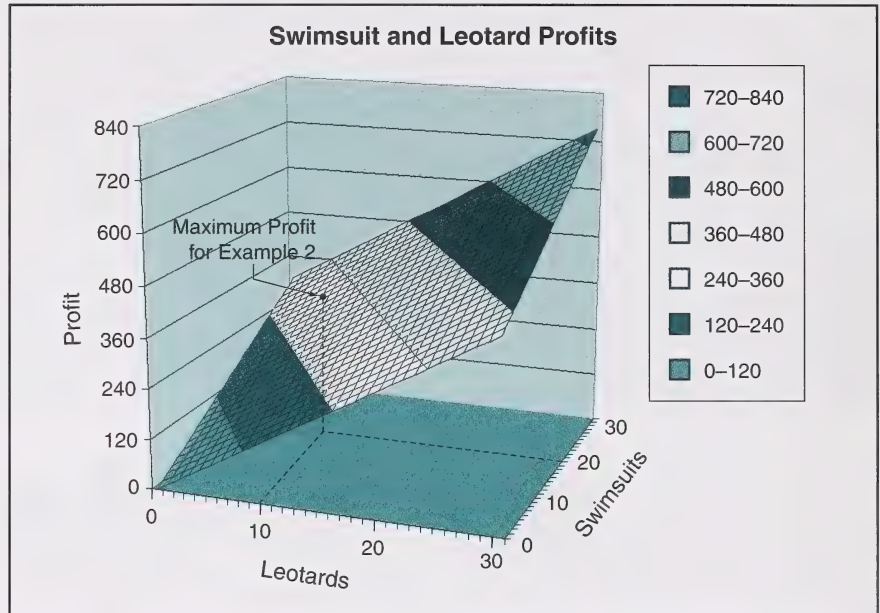
1. Answer exercises 2 to 10 of “Investigation: Maximizing Profits” on pages 200 and 201 of the textbook. Use the following graph to complete the investigation.



Compare your responses with the suggested answers in the Appendix, Activity 5, pages 77–82.

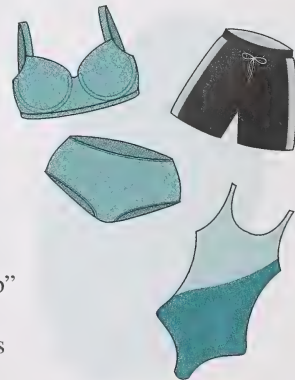


The following 3-D graph shows the profit picture for the manufacture of swimsuits and leotards as described in “Example 2: Manufacturing Sportswear.” Find the point on the base of the chart corresponding to the number of swimsuits and leotards produced. The profit is shown by the height (and colour) of the striped surface directly above the point you located in the base. Follow the dotted lines from 20 swimsuits and 10 leotards, up to the coloured plane.



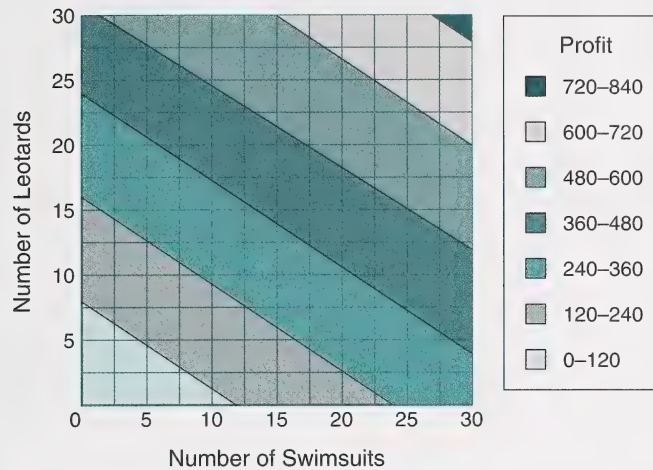
The surface is a visual representation of an objective function. That's the function for which you want to find the best value. The surface is flat because this objective function is linear.

In this graph, the number of leotards and the number of swimsuits are not bound by the constraints in “Example 2: Manufacturing Sportswear” (page 192 of the textbook). That is, the profits here go up to 30 swimsuits and 30 leotards, which is beyond the capability of the manufacturer. Nevertheless, the graph does suggest that the greatest profit within a restricted region will occur away from the origin.



The profit picture can also be represented by a “contour map” on a coordinate plane. In effect, you push the 3-D representation onto its base (and view it from above). This is the perspective you used in the previous investigation.

Swimsuit and Leotard Profits



Turn to page 201 of the textbook and read the bottom half of the page.



If linear objective functions always have their minimum or maximum at one of the vertices of the solution region, that means there are only a few points to check.



Turn to page 202 of the textbook and read the top half of the page; then work through “Example 1: Minimizing Vitamin Costs” on pages 202 to 204. Look for the five steps for solving optimization problems in the example. **Note:** The features from the CALCULATE menu can be used to find all the vertices. Use the Intersect feature to find the point of intersection of the two boundary lines, and use the Value feature to find the y-intercept for each equation.





Now try a similar question on your own. You may do hand-drawn graphs on a grid, or you may use your graphing calculator to complete the graphs.

2. Answer exercise 5 of “Exercises: Checking Your Skills” on page 208 of the textbook.

Compare your response with the suggested answer in the Appendix, Activity 5, pages 82–83.

You have just looked at problem situations in which you needed to determine the minimum value of an objective function. Finding the maximum value also involves the same five steps for solving optimization problems. Look for these steps in the following example.



Turn to pages 204 to 206 of the textbook and work through “Example 2: Maximizing Revenue.”

3. Answer exercises 1, 2, 3, 4, and 7 of “Exercises: Checking Your Skills” on pages 207 and 208 of the textbook. In exercise 1, assume the optimal value for the objective function is the maximum value.

Compare your responses with the suggested answers in the Appendix, Activity 5, pages 83–89.

LOOKING BACK

In this activity you used linear programming to find optimal solutions for minimum and maximum problems. You found that the optimal values the objective function appear at vertices of the solution region.



4. Turn to page 206 of the textbook and answer exercise 1 of “Discussing the Ideas.”
5. Turn to page 209 of the textbook and answer “Communicating the Ideas.”

Compare your responses with the suggested answers in the Appendix, Activity 5, page 89.

Follow-up Activities



This module dealt with Chapter 4: Linear Programming in the *Addison-Wesley Applied Mathematics 11 Source Book*.

Turn to page 212 of the textbook and take note of the skills and concepts listed for each of the tutorials. Also, read the column of important results and formulas you discovered. The table indicates concisely what you should know and be able to do after participating in the learning activities of this module.

The next question provides an opportunity for you to apply several concepts and skills at the same time.

Turn to page 215 of the textbook and answer exercise 9 of Part B of “What Should I Be Able to Do?”

Compare your response with the suggested answer in the Appendix, Follow-up Activities, pages 90–92.

If you had difficulties understanding the skills and concepts in Module 4: Linear Programming, it is recommended that you do the Extra Help. If you have a clear understanding of the skills and concepts in this module, it is recommended that you do the Enrichment. You may decide to do both.

EXTRA HELP

Success in this module depends on your ability to identify the half-plane that corresponds to an inequality. Suppose you are about to identify the appropriate half-plane by shading. You can use a test point to determine which half-plane to shade.

Pick a test point anywhere on the coordinate plane, but not on the boundary line itself; then substitute its coordinates into the inequality. If the coordinates of the test point satisfy the inequality, the half-plane on the same side as the test point should be shaded. If the coordinates do not satisfy the inequality, the half-plane on the other side of the boundary line should be shaded.



Certain points have coordinates for which the inequality can be easily checked. These points, when used as test points, are called **convenient test points**. The point $(0, 0)$ can often serve as a convenient test point.

Example

Graph $4x - 5y \leq 30$ on a coordinate plane.

Solution

Step 1: Graph the corresponding equation, $4x - 5y = 30$. For this equation, use the intercepts to graph the line.

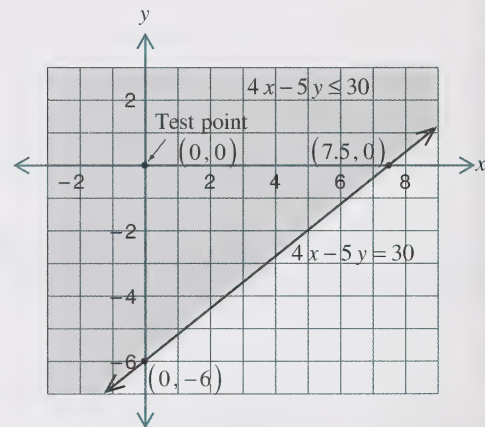
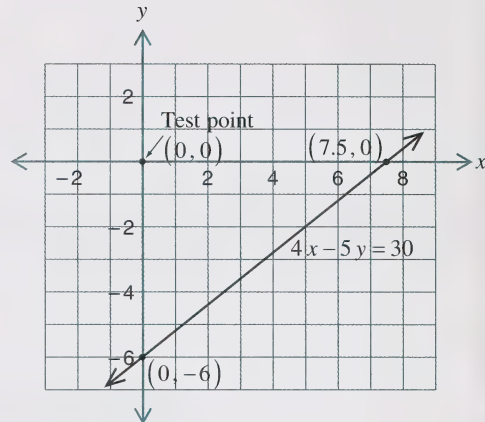
Step 2: Pick a convenient test point, like $(0, 0)$, on one side of the line, but not on the line. See if it satisfies the inequality.

LS	RS
$4x - 5y$	30
$= 4(0) - 5(0)$	
$= 0$	
LS	≤ RS

The test point satisfies the inequality.

Step 3: Shade the appropriate half-plane.

Because the test point satisfies the inequality and is above the boundary line, the upper half-plane should be shaded.



Sketch the following inequalities on a coordinate plane.

1. $2x \leq y + 1$

2. $3x - y > 0$

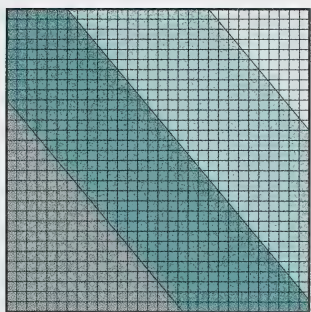
Compare your responses with the suggested answers in the Appendix, Follow-up Activities: Extra Help, pages 92–93.

Another Dimension

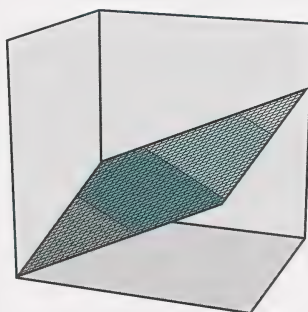


The following graphs show a picture much like the profit picture for the manufacture of swimsuits and leotards as described in “Example 2: Manufacturing Sportswear” on page 192 of the textbook. The graph on the left is the 2-D contour representation (much like the graph you studied in Activity 5). The graph on the right shows the 3-D version. As you were shown in Activity 5, the height of the surface above any point in the base of the chart in the 3-D version indicates the profit for a certain combination of swimsuit and leotard production figures. The straight lines on the 2-D version and the flat shape of the 3-D version are typical for linear objective functions.

Linear Object Function



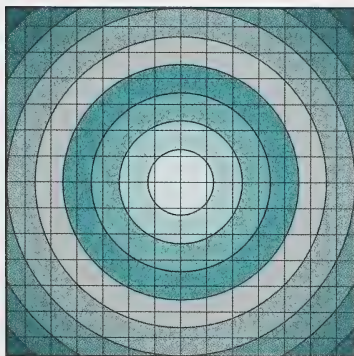
Linear Object Function



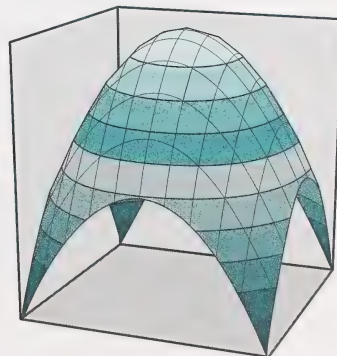
Non-Linear Functions

The following graphs depict a non-linear objective function. Notice the concentric circles forming the contours in the 2-D graph on the left and the rounded shape of the surface in the 3-D graph on the right. This is similar to the contour lines on a contour map, which you studied in Module 1 of this course.

Non-Linear Object Function



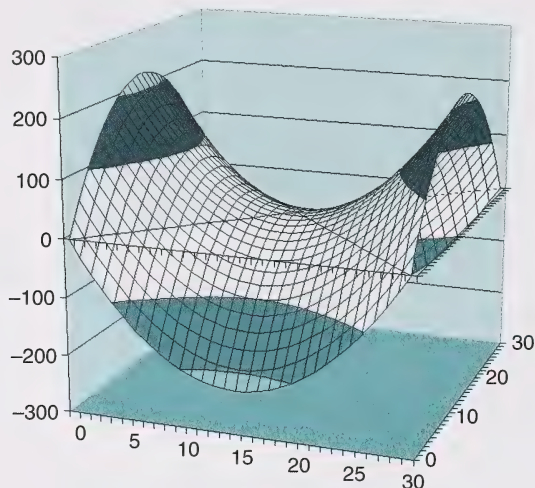
Non-Linear Object Function



1. Describe or circle the maximum point in each graph of the non-linear object function.
2. Explain why the maximum (or minimum) of a linear function on a straight-sided solution region is easier to find than the maximum (or minimum) of a non-linear function with the same solution region.

Compare your responses with the suggested answers in the Appendix, Follow-up Activities: Enrichment, page 94.

The following is a 3-D depiction of another type of non-linear function.



You can make a graph like this using a spreadsheet program, like Microsoft® *Excel*.

Step 1: Start a table of data (as shown). Continue the series 1, 2, 3, ..., 30 along the top and down the left side of the table.

	A	B	C	D	E	F	G	H
1		0	1	2	3	4	5	6
2	0							
3	1							
4	2							
5	3							

Step 2: Enter the following expression in cell B2.

$$=-(\text{POWER}(\$A2-15, 2) - \text{POWER} (B\$1-15, 2))$$

This expression is equivalent to $-\left[(x-15)^2 - (y-15)^2\right]$.

Step 3: Fill this expression across row 2. Then fill down to complete the table.

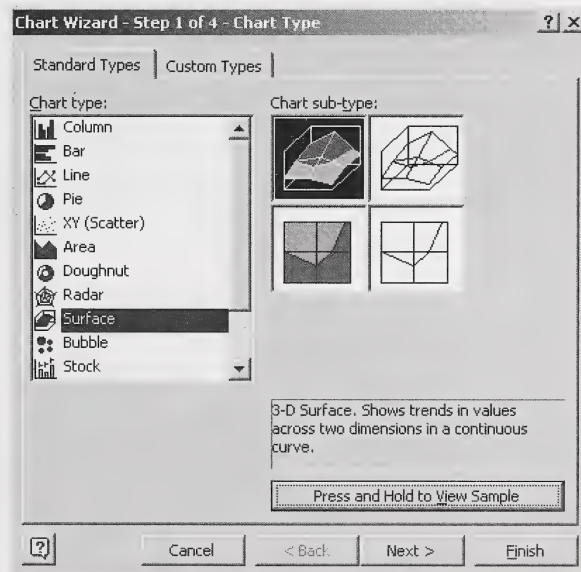
	A	B	C	D	E
1		0	1	2	3
2	0	0	-29	-56	-81
3	1	29	0	-27	-52
4	2	56	27	0	-25
5	3	81	52	25	0

AC	AD	AE	AF
27	28	29	30
-81	-56	-29	0
-52	-27	0	29
-25	0	27	56
0	25	52	81

29	27	81	52	25	0
30	28	56	27	0	-25
31	29	29	0	-27	-52
32	30	0	-29	-56	-81

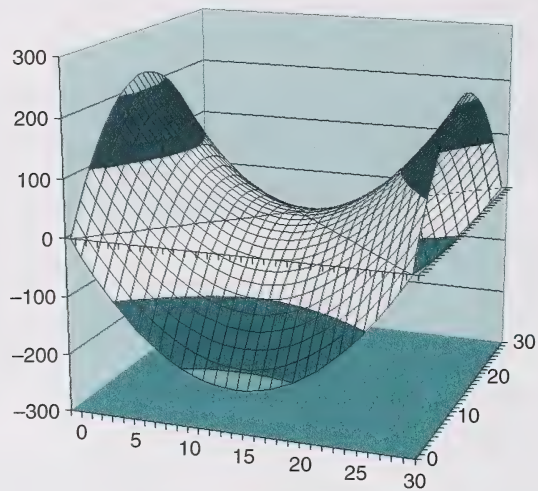
	0	25	52	81
-25	0	27	56	
-52	-27	0	29	
-81	-56	-29	0	

Step 4: Activate the chart wizard. Select Surface as the Chart Type and 3-D Surface as the Chart sub-type.

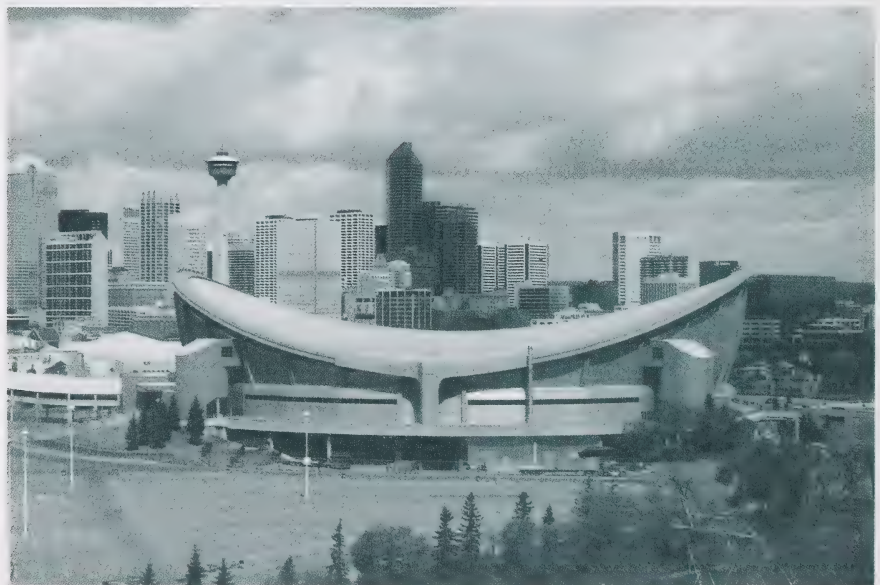


MICROSOFT CANADA CORPORATION

Step 5: Press “Finish” to obtain the graph. **Note:** Adjust the scales of the axes to obtain the following graph.



You may have figured out that the graph you made is shaped like the roof of a well-known building in Calgary—the Pengrowth Saddledome. You see, 3-D representations of equations can provide a model for real-world objects.



Completing the Project

At the beginning of this module you were introduced to the requirements for your Module 4 project, Trail Mix. You should now have done a significant part of your research for this project, and you should have found answers for the questions posed in the project introduction on page 168 of the textbook.

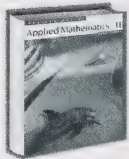
Turn to page 198 of the textbook and read the introductory paragraphs of “Finding Feasible Snack Mixes.”

1. Answer exercise 1 of “Finding Feasible Snack Mixes” on page 198 of the textbook.

Compare your response with the suggested answer in the Appendix, Module Project, page 94.

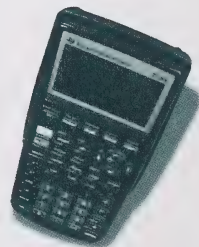
You may want to refer to the Diet & Nutrition Calculators at the following Internet site:

<http://www.drkoop.com/tools/calculator/#>



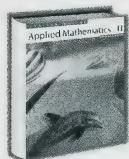
Refer to exercise 2 of “Finding Feasible Snack Mixes” on page 198 of the textbook and to the chart on page 199 as you do the following exercises. **Note:** The chart is for 100 g of each item.

2. Propose amounts of semisweet chocolate and seedless raisins that meet the conditions described in exercise 2 of “Finding Feasible Snack Mixes.” Choose amounts of each ingredient, and check that your potential trail mix’s fat content is reduced by 25% and its fibre, carbohydrates, or protein is enriched by 5%. You may have to adjust the amounts. Show your calculations.
3. Let x and y be the masses (in grams) of the ingredients making up a mixture meeting the first two bulleted conditions in exercise 2 of “Finding Feasible Snack Mixes.” These conditions apply to any mixture you might propose.
 - a. Write a system of 3 linear inequalities to represent the first two bulleted conditions (constraints).
 - b. Sketch a graph to show the solution region representing the possible make-up of the trail mixture according to just the first two bullets.
4. Let x be the mass (in grams) of semisweet chocolate and y be the mass (in grams) of seedless raisins making up the trail mix. In the chart, each has a unique value indicating the relative amount of fat. These values will become coefficients of x and y in another condition (constraint).
 - a. Write an inequality in terms of x and y to represent the third bulleted condition. Rewrite the condition in $Y =$ form so you can enter the inequality into your calculator.
 - b. Use your graphing calculator to graph the solution region for the system of equations corresponding to the first three bulleted conditions in exercise 2 of “Finding Feasible Snack Mixes.” (You determined some of the inequalities in question 3.) Apply reverse shading on your calculator. **Note:** Keep the graph in your calculator; you will need it for question 6.
 - c. Identify the solution you found in exercise 2.
 - d. From the solution region, pick two other solutions that meet the condition stated in the fourth bullet. You may do this by trial and error or by adding an additional inequality to your system of inequalities and selecting from the resulting (possibly smaller) solution region. Check your solution.



Compare your responses with the suggested answers in the Appendix, Module Project, pages 94–99.

You now have three compositions for the trail mix of semisweet chocolate and seedless raisins. Knowing the cost of ingredients allows you to pick the **best** trail mix of semisweet chocolate and seedless raisins based on price. Here, the best trail mix is the least expensive one.



Turn to page 210 of the textbook and refer to the table showing the price of each ingredient. The information is needed to determine an objective function based on costs.

5. Find the total cost of each of the three feasible compositions you determined for trail mixes of semisweet chocolate and seedless raisins. The formula for the cost (in cents), based on the table on page 210, is $C = 21\left(\frac{x}{100}\right) + 10\left(\frac{y}{100}\right)$.
6. Again, let x be the mass (in grams) of semisweet chocolate and y be the mass (in grams) of seedless raisins in a mixture of trail mix. Use the objective function from exercise 5 and your graph of the solution region of the system of inequalities (from exercise 4.b.) to do the following.
 - a. Determine the least-expensive composition of a trail mix consisting of semisweet chocolate and seedless raisins. What is the cost of the least-expensive composition of trail mix?
 - b. Compare the nutritional content of protein, total fat, carbohydrate, and dietary fibre of this least-expensive trail mix to the nutritional content of the company's mix. Turn to page 198 of the textbook and refer to the chart indicating the nutritional content of the company's present trail mix.
7. Turn to pages 216 and 217 of the textbook and answer exercises 10 and 11 of Part C, of "What Should I Be Able to Do?" In exercise 10, the student's work is based on the student's response to exercise 2 of "Finding Feasible Snack Mixes" on page 198 of the textbook. In exercise 11, the student's work is based on the student's response to "Determining the Best Trail Mix" on page 210 of the textbook.
8. Turn to page 218 of the textbook and read the student's recommendation. What did the student feel was the most important factor in choosing the best trail mix?



Compare your responses with the suggested answers in the Appendix, Module Project, pages 99–101.

The Module Project

Now that you have more insight into what is required for the module project, turn to your Project Booklet and complete the module project, Trail Mix.

Submit your completed Module 4 Project Booklet to your teacher.



Module Summary

In this module you graphed linear inequalities on a coordinate plane. You solved systems of inequalities and used these systems to model and solve problem situations. You also applied systems of inequalities to find optimal solutions.



Did you know that a network of paths crossing Canada has been designated as the Trans Canada Trail. The Trans Canada Trail is for recreational use only (walking, cycling, cross-country skiing, horseback riding, snowmobiling, and so on).

In your trail mix project, you learned to put together the best mix in terms of price and nutrition. A trail mix you make could one day help to keep up your stamina on the Trans Canada Trail.

Module Assignment

To demonstrate what you have learned in this module, complete the module assignment in the Assignment Booklet.



**Submit your completed Module 4 Assignment Booklet
to your teacher.**

Glossary
Suggested Answers
Image Credits



GLOSSARY

constraint: a condition limiting the range of values

Constraints are often used in an inequality.

half-plane: the region on one side of a line on a coordinate plane including or excluding the line itself

linear equation: an equation whose graph is a straight line; such an equation can be written in the form $y = mx + b$

The variables of a linear equation are in the first degree.

linear inequality: a statement in which one quantity is greater than or less than another quantity

objective function: a function whose optimal value for points of a solution region is required

The optimal value is usually the maximum or minimum value of the function (in the solution region).

optimal solution: a solution for which the objective function is either a maximum or a minimum value

solution of a system of inequalities: an ordered pair that satisfies all the inequalities of the system

solution region: a region of the coordinate plane consisting of solutions of a system of inequalities

system of linear inequalities: a group of linear inequalities that are to be considered at the same time

SUGGESTED ANSWERS

Activity 1: The Graph of a Linear Inequality

1. Textbook exercises 1, 2, 3.d., and 4.e. of “Practise Your Prior Skills,” p. 170

1. a. Check $x = 6$.

LS	RS
$2x - 5$	11
$= 2(6) - 5$	
$= 12 - 5$	
$= 7$	
LS	< RS

Check $x = 8$.

LS	RS
$2x - 5$	11
$= 2(8) - 5$	
$= 16 - 5$	
$= 11$	
LS	$\nless$ RS

Check $x = 10$.

LS	RS
$2x - 5$	11
$= 2(10) - 5$	
$= 20 - 5$	
$= 15$	
LS	$\nless$ RS

Of the given numbers, $x = 6$ is a solution.

b. Check $x = 2$.

LS	RS
$3x+1$	10
$=3(2)+1$	
$=6+1$	
$=7$	
LS	RS

$\geq$

Check $x = 3$.

LS	RS
$3x+1$	10
$=3(3)+1$	
$=9+1$	
$=10$	
LS	RS

$\geq$

Check $x = 4$.

LS	RS
$3x+1$	10
$=3(4)+1$	
$=12+1$	
$=13$	
LS	RS

$\geq$

Of the given numbers, $x = 3$ and $x = 4$ are solutions.

2. a. $x \geq 2$ b. $x < 3$ c. $x > -1.5$ or $x > -1\frac{1}{2}$



4. e. $2x+5 < 3x-1$

$-x+5 < -1$ $\leftarrow$ Subtract $3x$ from both sides.

$-x < -6$ $\leftarrow$ Subtract 5 from both sides.

$x > 6$ $\leftarrow$ Multiply through by -1 , and switch the inequality symbol.

Check

Try a number that is in the solution set, such as $x = 8$.

LS	RS
$2x+5$	$3x-1$
$=2(8)+5$	$=3(8)-1$
$=16+5$	$=24-1$
$=21$	$=23$
LS	RS

$<$

Since $x = 8$ satisfies the original inequality, the solution is $x > 6$.

Activity 1 (continued)

2. Textbook exercises 5.a. and 6.a. of “Practice Your Prior Skills,” pp. 170 and 171

5. a. $2x + 3y - 6 = 0$

$$3y - 6 = -2x \quad \leftarrow \text{Subtract } 2x \text{ from both sides.}$$

$$3y = -2x + 6 \quad \leftarrow \text{Add 6 to both sides.}$$

$$y = -\frac{2}{3}x + 2 \quad \leftarrow \text{Divide both sides by 3.}$$

slope $\uparrow$ $\uparrow$ y-intercept

The slope is $-\frac{2}{3}$, and the y-intercept is $(0, 2)$.

6. a. $2x - 3y + 6 = 0$

$$2x - 3y = -6$$

Use the x- and y-intercepts to graph the equation.

Let $x = 0$.

Let $y = 0$.

$$\therefore 2x - 3y = -6$$

$$\therefore 2x - 3y = -6$$

$$2(0) - 3y = -6$$

$$2x - 3(0) = -6$$

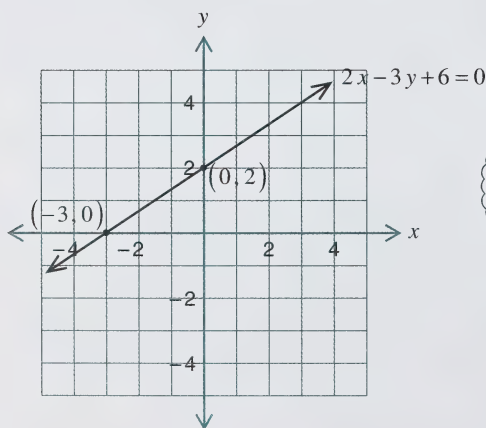
$$-3y = -6$$

$$2x = -6$$

$$y = 2$$

$$x = -3$$

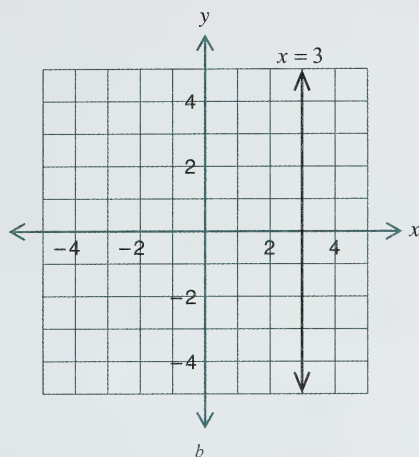
The y-intercept is at $(0, 2)$, and the x-intercept is at $(-3, 0)$.



You could have also graphed the line using a table of values.

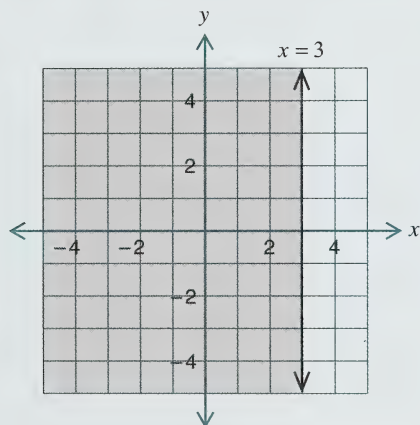
3. Textbook exercises 1 to 10 of “Investigation 1: Graphing Half-Planes,” p. 171

1. and 2.



3. Yes, all of the points on the line satisfy the equation $x = 3$.

4.

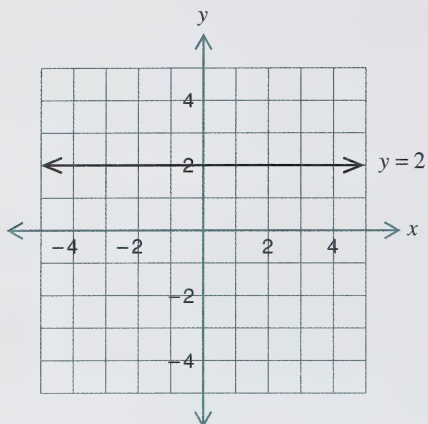


All points have x -coordinates that are less than 3.

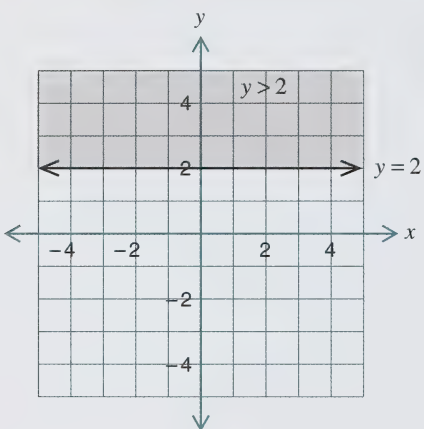
5. All points to the right of the line $x = 3$ have x -coordinates that are greater than 3.

Activity 1 (continued)

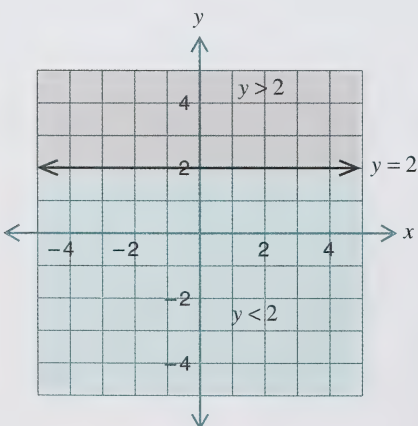
6.



7.



8.



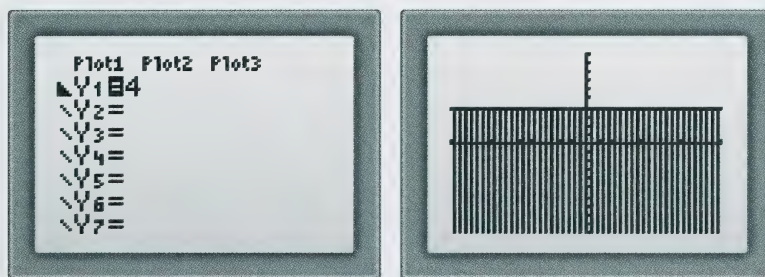
9. To draw the graph of $x > 4$ on a grid, shade the region to the right of the vertical line $x = 4$.

The points on the line $x = 4$ do not satisfy the inequality $x > 4$. You could draw the line in a (different) colour or you could draw the line as a dotted line.

10. To graph $y \leq 1$ on a grid, graph the horizontal line $y = 1$ as a solid line (the usual way) and shade the region below the line. Making the line solid can be interpreted as showing that the points on the line are included in the graph.

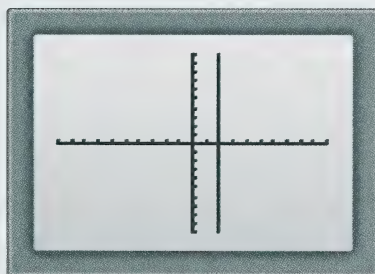
4. Textbook exercises 1 and 2 of “Discussing the Ideas,” pp. 172 and 173

1. Enter the corresponding equation, $y = 4$, into the equation editor. Then select the Below icon as the graph style. This style will shade the region below the line $y = 4$. Press **GRAPH** to display the graph of the inequality.



2. To draw a vertical line corresponding to $x = 2$ first select the standard window settings, **ZOOM** **6**; then press the following.

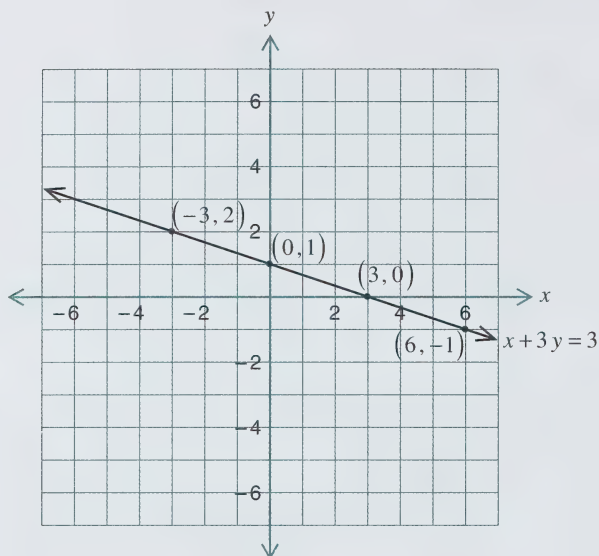
CLEAR **2nd** **[DRAW]** **4** (4:Vertical) **2**



Activity 1 (continued)

5. Textbook exercises 1 to 10 of “Investigation 2: Graphing an Inequality in Two Variables,” p. 172

1. to 3.



From left to right, the four points are $(-3, 2)$, $(0, 1)$, $(3, 0)$, and $(6, -1)$.

4. Check $(-3, 2)$.

LS	RS
$x + 3y$	3
$= (-3) + 3(2)$	
$= -3 + 6$	
$= 3$	
LS	RS

Check $(0, 1)$.

LS	RS
$x + 3y$	3
$= 0 + 3(1)$	
$= 0 + 3$	
$= 3$	
LS	RS

Check $(3, 0)$.

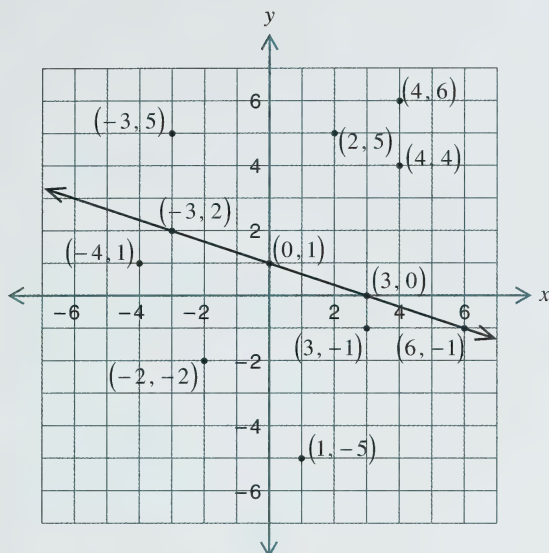
LS	RS
$x + 3y$	3
$= 3 + 3(0)$	
$= 3 + 0$	
$= 3$	
LS	RS

Check $(6, -1)$.

LS	RS
$x + 3y$	3
$= 6 + 3(-1)$	
$= 6 - 3$	
$= 3$	
LS	RS

All the points satisfy the inequality.

5.



6. Check $(-3, 5)$.

LS	RS
$x + 3y$	3
$= -3 + 3(5)$	
$= -3 + 15$	
$= 12$	
LS	$\geq$ RS

Check $(2, 5)$.

LS	RS
$x + 3y$	3
$= 2 + 3(5)$	
$= 2 + 15$	
$= 17$	
LS	$\geq$ RS

Check $(4, 4)$.

LS	RS
$x + 3y$	3
$= 4 + 3(4)$	
$= 4 + 12$	
$= 16$	
LS	$\geq$ RS

Check $(4, 6)$.

LS	RS
$x + 3y$	3
$= 4 + 3(6)$	
$= 4 + 18$	
$= 22$	
LS	$\geq$ RS

These four points should be circled on your grid.

Activity 1 (continued)

Check $(-4, 1)$.

LS	RS
$x + 3y$	3
$= -4 + 3(1)$	
$= -4 + 3$	
$= -1$	
LS	$\nlessgtr$ RS

Check $(-2, -2)$.

LS	RS
$x + 3y$	3
$= -2 + 3(-2)$	
$= -2 - 6$	
$= -8$	
LS	$\nlessgtr$ RS

Check $(1, -5)$.

LS	RS
$x + 3y$	3
$= 1 + 3(-5)$	
$= 1 - 15$	
$= -14$	
LS	$\nlessgtr$ RS

Check $(3, -1)$.

LS	RS
$x + 3y$	3
$= 3 + 3(-1)$	
$= 3 - 3$	
$= 0$	
LS	$\nlessgtr$ RS

The points from exercise 5 that satisfy the inequality are $(-3, 5)$, $(2, 5)$, $(4, 4)$, and $(4, 6)$.

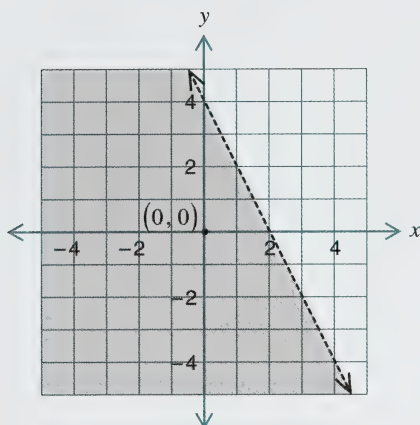
- All points satisfying the inequality $x + 3y \geq 3$ are on or above the line.
- The points below the line would satisfy the inequality $x + 3y \leq 3$.
- Solve the inequality for y , and graph the corresponding linear equation.

If the inequality includes equality ($\leq$ or $\geq$), the line is part of the graph. If the inequality excludes equality ($<$ or $>$), the line is not part of the graph.

If $y \geq f(x)$ or $y > f(x)$, shade the half-plane above the boundary line.

If $y \leq f(x)$ or $y < f(x)$, shade the half-plane below the boundary line.

10. Based on the prediction in exercise 8, the points below the boundary line, $2x + y = 4$, will satisfy $2x + y < 4$.



Check $(0, 0)$.

LS	RS
$2x + y$	4
$= 2(0) + 0$	
$= 0$	
LS	< RS

Since a test point below the line satisfies the inequality, the half-plane below the line is the solution and should be shaded to show that these points will satisfy the inequality.

6. Textbook exercises 3 to 6 of “Discussing the Ideas,” p. 173

- When the coordinates of any point from the region below the line are substituted into the inequality $2x + y < 4$, the result will be less than 4. That is, any point from the region below the line satisfies the inequality.
- Draw a dotted line for the equation $2x + y = 4$.
- You would replace the dotted line with a solid line.
- Choose a point above or below the line as a test point. Substitute the coordinates of the test point into the inequality. If the inequality holds with the substituted values, the half-plane on the same side of the test point should be shaded. If the inequality fails to hold, the half-plane on the other side of the boundary line should be shaded.

Activity 1 (continued)

7. Textbook exercise “Communicating the Ideas,” p. 173

Answers will vary. A sample explanation is given.

When graphing a linear inequality, you must first graph the corresponding linear equation. The graph of this equation divides the coordinate plane into two half-planes. One half-plane is a solution of the inequality, and the other is not a solution. Therefore, your success in graphing the linear inequality hinges on being able to graph the related linear equation quickly and identify which half-plane satisfies the inequality.

Activity 2: Graphing a Linear Inequality in Two Variables

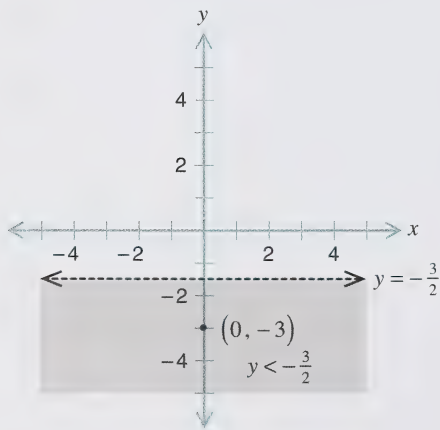
1. The last step should be to apply a test point. The purpose of this step is to decide which half-plane should be shaded or to confirm that the correct half-plane is shaded.

2. Textbook exercises 1 and 4 of “Discussing the Ideas,” p. 178

1. It is linear because it contains variables of degree one. It is an inequality because it has the $\leq$ sign rather than the equal sign. It is in two variables because there are two different variables: x and y .

4. Wilma will graph the inequality as an arrow on a number line, whereas Frank will graph it as a half-plane on a coordinate grid.

3.



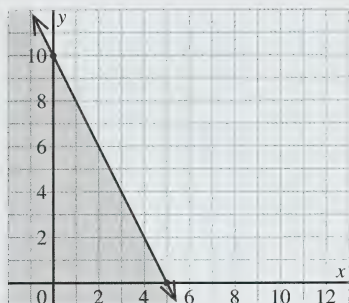
Check using $(0, -3)$.

LS	RS
y	$-\frac{3}{2}$
$= -3$	
LS	< RS

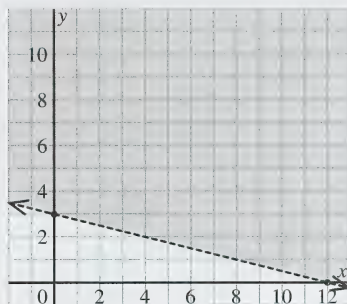
4. Textbook exercises 2, 3.d., 3.f., 3.h., 4.b., 4.d., 5.b., and 6.b. of “Exercises: Checking Your Skills,” pp. 178 and 179

2. Note that the dots on the axes are the x - and y -intercepts. Draw the boundary lines through these points.

a. $2x + y \leq 10$



b. $x + 4y > 12$

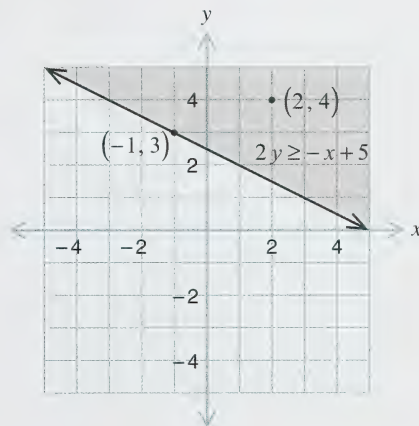


3. d. Solve the inequality for y .

$$2y \geq -x + 5$$

$$y \geq -\frac{1}{2}x + \frac{5}{2} \quad \leftarrow \text{Divide both sides by 2.}$$

Graph the corresponding equation, $y = -\frac{1}{2}x + \frac{5}{2}$, and shade the appropriate half-plane. In the inequality, the $\geq$ sign indicates that the boundary line is solid and that the half-plane above the boundary line is shaded.



Check $(-1, 3)$.

Check $(2, 4)$.

LS	RS
$2y$	$-x + 5$
$= 2(3)$	$= -(-1) + 5$
$= 6$	$= 1 + 5$
	$= 6$
LS	$\geq$ RS

LS	RS
$2y$	$-x + 5$
$= 2(4)$	$= -(2) + 5$
$= 8$	$= 3$
LS	$\geq$ RS

Activity 2 (continued)

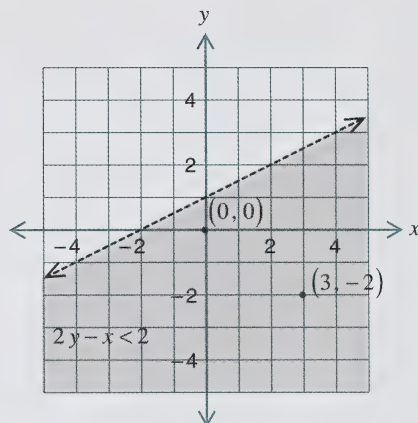
- f. Solve the inequality for y .

$$2y - x < 2$$

$$2y < x + 2 \quad \leftarrow \text{Add } x \text{ to both sides.}$$

$$y < \frac{1}{2}x + 1 \quad \leftarrow \text{Divide both sides by 2.}$$

Graph the corresponding equation, $y = \frac{1}{2}x + 1$, and shade the appropriate half-plane. In the inequality, the $<$ sign indicates that the boundary line is dotted and that the half-plane below the boundary line is shaded.



Check $(0, 0)$.

LS	RS
$2y - x$	2
$= 2(0) - 0$	
$= 0 - 0$	
$= 0$	
LS	< RS

Check $(3, -2)$.

LS	RS
$2y - x$	2
$= 2(-2) - 3$	
$= -4 - 3$	
$= -7$	
LS	< RS

- h. $4x - 5y + 20 \leq 0$

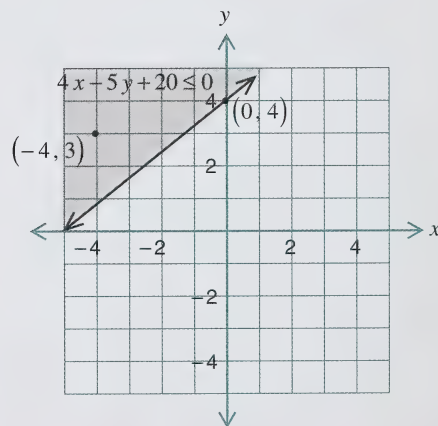
$$4x + 20 \leq 5y$$

$$5y \geq 4x + 20$$

$$y \geq \frac{4}{5}x + 4$$

Notice that the inequality sign changes when the two sides of the inequality are interchanged.

In the inequality, the $\geq$ sign indicates that the boundary line is solid and that the half-plane above the boundary line is shaded.



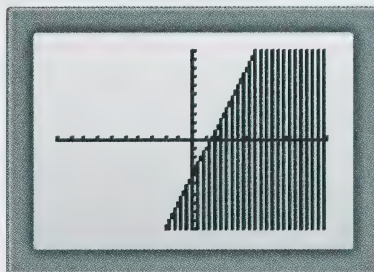
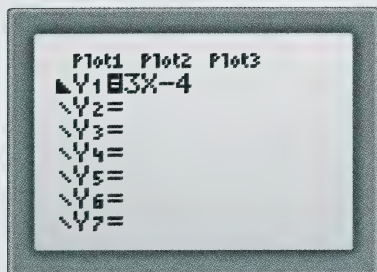
Check $(-4, 3)$.

LS	RS
$4x - 5y + 20$	0
$= 4(-4) - 5(3) + 20$	
$= -16 - 15 + 20$	
$= -11$	
LS	RS

Check $(0, 4)$.

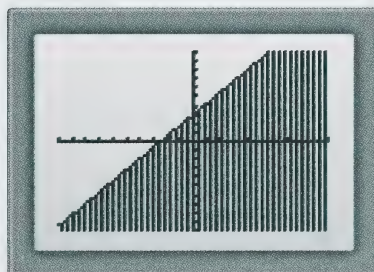
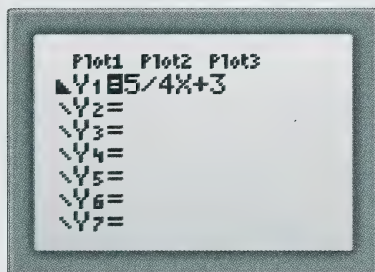
LS	RS
$4x - 5y + 20$	0
$= 4(0) - 5(4) + 20$	
$= 0 - 20 + 20$	
$= 0$	
LS	RS

4. b.



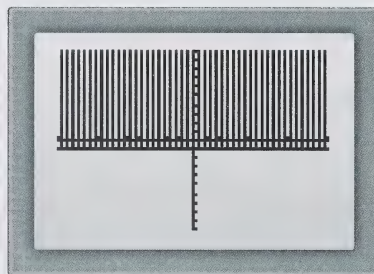
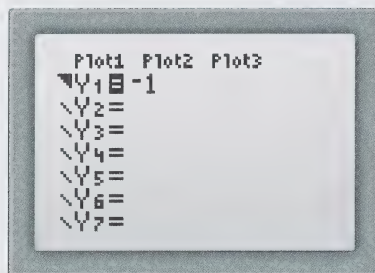
d. $4y < 5x + 12$

$$y < \frac{5}{4}x + 3$$

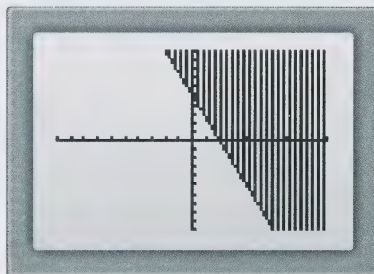
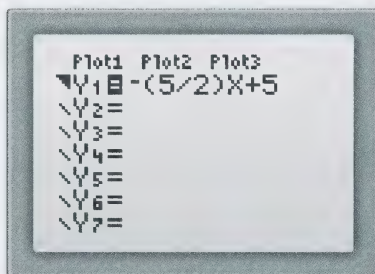


Activity 2 (continued)

5. b. $y+1>0$
 $y>-1$



6. b. $5x+2y>10$
 $2y>-5x+10$
 $y>-\frac{5}{2}x+5$



5. Textbook exercises 2 and 3 of “Discussing the Ideas,” p. 178

- The test point $(0, 0)$ is a convenient test point; when the coordinates $x = 0$ and $y = 0$ are substituted into the inequality, the calculations are very simple.

When the boundary line goes through the point $(0, 0)$, another test point must be used.

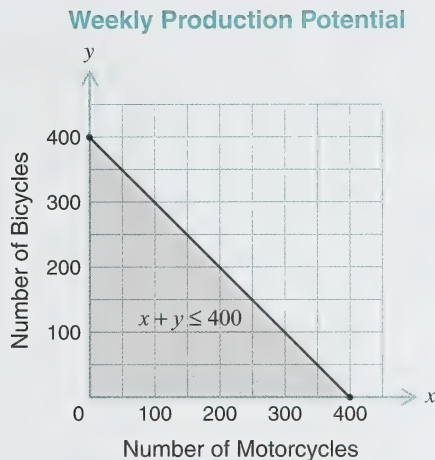
- You should use a broken line because the boundary line itself is not part of the solution. Any point on the line would make the left side of the inequality equal to the right side rather than making the left side strictly less than the right side.

6. Textbook exercises 7 and 8 of “Exercises: Checking Your Skills,” p. 179

7. a. The wording *no more than 400* means less than or equal to 400.
- b. The variables x and y are discrete variables because they stand for whole numbers of items only. You can’t have a partial motorcycle or a partial bicycle.

c. $x + y \leq 400$

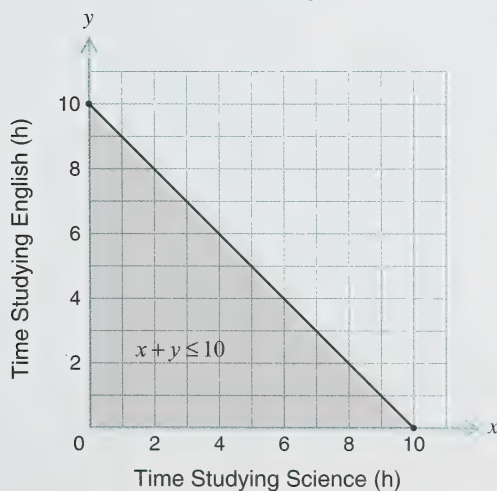
$$y \leq -x + 400$$



Note: Some inequalities involve **discrete** variables. If the individual points satisfying the inequality are very close together on the grid, you may use shading to represent these points. If you do use shading to represent discrete points, be sure to indicate how someone should interpret the shading.

8. a. The variables x and y are continuous variables because they represent time, which is a continuous quantity.

b. **Potential Study Times**



Activity 2 (continued)

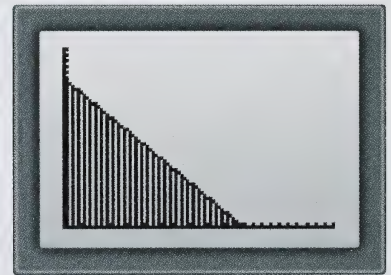
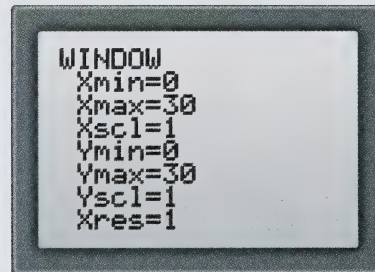
7. a. Let x represent the number of cakes that require 125 g (0.125 kg) of flour and y represent the number of cakes that require 100 g (0.100 kg) of flour.

$$\therefore 0.125x + 0.100y \leq 2.5$$

b. $0.125x + 0.100y \leq 2.5$

$$0.100y \leq -0.125x + 2.5$$

$$y \leq -1.25x + 25$$



- c. Feasible solutions to the problem can be obtained by selecting whole-numbered points within the shaded region of the graph.
- d. Answers will vary. Two possible solutions are $(5, 10)$ and $(15, 5)$.

Check $(5, 10)$.

LS	RS
$0.125x + 0.100y$	2.5
$= 0.125(5) + 0.100(10)$	
$= 0.625 + 1.000$	
$= 1.625$	
LS	RS

Check $(15, 5)$.

LS	RS
$0.125x + 0.100y$	2.5
$= 0.125(15) + 0.100(5)$	
$= 1.875 + 0.500$	
$= 2.375$	
LS	RS

Sarah has enough flour to make 5 cakes requiring 125 g of flour and 10 cakes requiring 100 g of flour, or 15 cakes requiring 125 g of flour and 5 cakes requiring 100 g of flour.

8. Advantages of using pencil and paper over using a graphing calculator are as follows:

- You can show shading above or below the boundary line and show whether the boundary line is a solid line (part of the solution) or a broken line (not part of the solution). A graphing calculator does not show whether the boundary line is part of the solution.
- When the variables are discrete rather than continuous, you can show individual points making up the solution. A graphing calculator can only show the region where the solutions can be found.

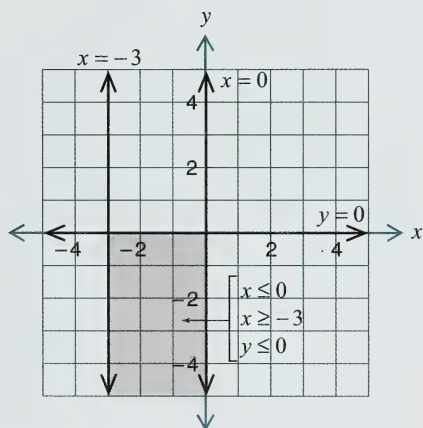
9. The graph of a linear inequality consists of a region on a coordinate plane. The graph of a linear equation consists of only a line.

Activity 3: The Solution of a System of Linear Inequalities

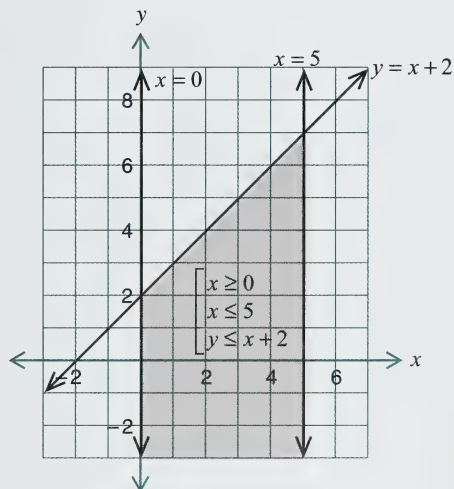
1. Textbook exercises 2, 3, 4.b., and 4.c. of “Exercises: Checking Your Skills,” pp. 187 and 188

2. a. $y \leq 2$
b. $x > -2$
c. $y < x$

3. a.

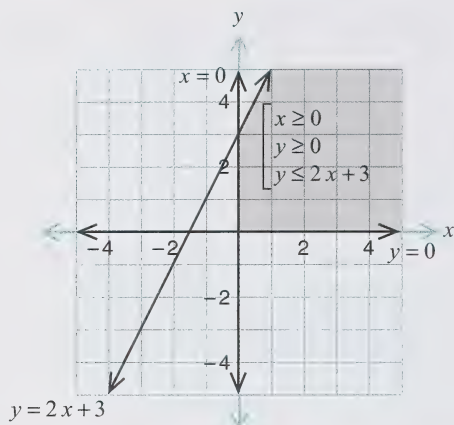


b.



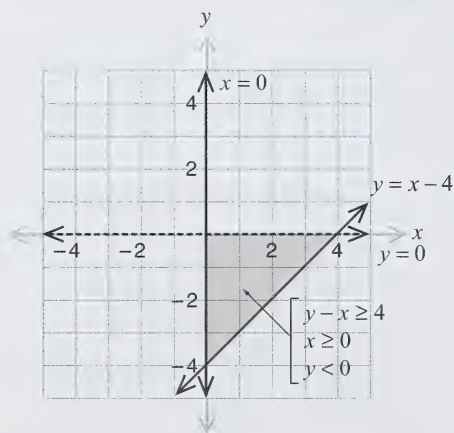
Activity 3 (continued)

c.



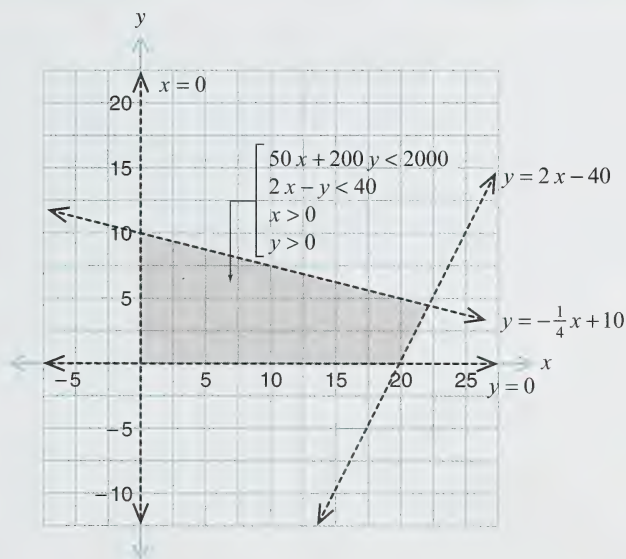
4. b.

Inequality	Boundary Line	Solution Region
$y - x \geq -4$ $y \geq x - 4$	$y = x - 4$	• solid boundary line • half-plane above boundary line shaded
$x \geq 0$	$x = 0$	• solid boundary line • half-plane to the right of boundary line shaded
$y < 0$	$y = 0$	• dotted boundary line • half-plane below boundary line shaded



c.

Inequality	Boundary Line	Solution Region
$50x + 200y < 2000$ $200y < -50x + 2000$ $y < -\frac{1}{4}x + 10$	$y = -\frac{1}{4}x + 10$	- dotted boundary line - half-plane below boundary line shaded
$2x - y < 40$ $-y < -2x + 40$ $y > 2x - 40$	$y = 2x - 40$	- dotted boundary line - half-plane above boundary line shaded
$x > 0$	$x = 0$	- dotted boundary line - half-plane to the right of boundary line shaded
$y > 0$	$y = 0$	- dotted boundary line - half-plane above boundary line shaded



2. Textbook exercise 1 of “Discussing the Ideas,” p. 186

- Region 3 represents the solution to the system of inequalities; it is the intersection of the two half-planes corresponding to the two inequalities of the system.

The points in regions 2 and 4 satisfy only one of the inequalities of the system. The points in region 1 do not satisfy either inequality of the system. Therefore, regions 1, 2, and 4 do not represent the solution of the system.

Activity 3 (continued)

3. Textbook exercise 5 of “Exercises: Checking Your Skills,” p. 188

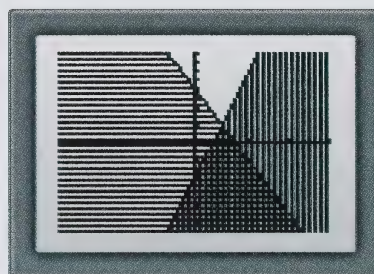
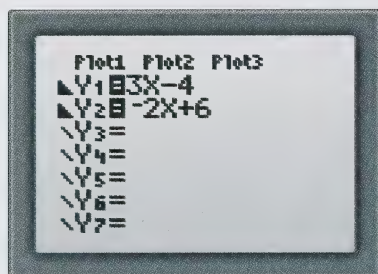
5. $3x - y > 4$

$$-y > -3x + 4$$

$$y < 3x - 4$$

$$2x + y \leq 6$$

$$y \leq -2x + 6$$

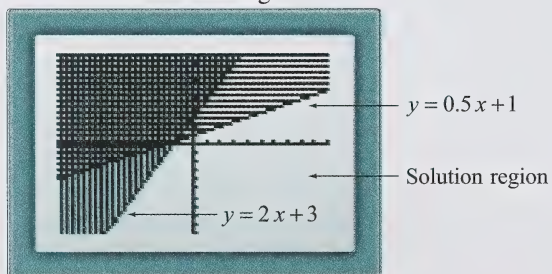


4. Textbook exercises 4 and 5 of “Discussing the Ideas,” p. 186

4. The inequality satisfied is $5x + 8y \geq 10$. The relation $\geq$ is the reverse of $\leq$. In reverse shading, the solution region of the reverse inequality is shaded.
5. With the inequality rearranged so that y is isolated on the left side, “greater than” ($>$) and “greater than or equal to” ($\geq$) correspond to shading above the boundary line. To shade above the boundary line, select the upper triangle shading. “Less than” ($<$) and “less than or equal to” ($\leq$) correspond to shading below the boundary line. To shade below the boundary line, select the lower triangle shading.

5. Textbook exercises 1, 6, and 7 of “Exercises: Checking Your Skills,” pp. 187 and 188

1. a. reverse shading

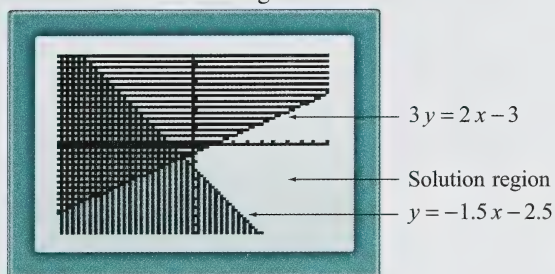


The solution region is below both boundary lines.

$$\therefore y < 2x + 3$$

$$y < 0.5x + 1$$

b. reverse shading



The solution region is above inequality 1 and below inequality 2.

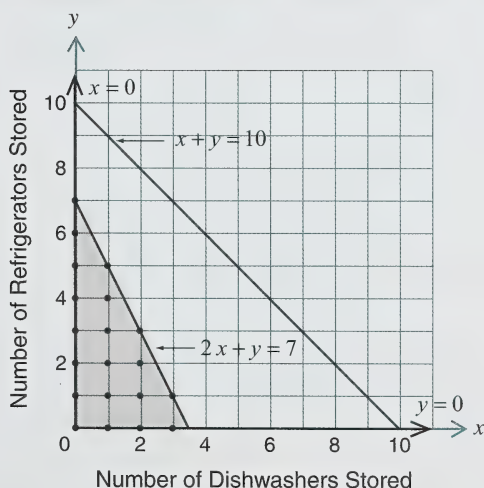
$$\therefore y > -1.5x - 2.5$$

$$3y < 2x - 3$$

6. a.

Inequality	Reason
$x \geq 0$	The number of dishwashers must be non-negative.
$y \geq 0$	The number of refrigerators must be non-negative.
$x + y \leq 10$	There is storage for only 10 appliances.
$2x + y \leq 7$	The time to unpack each dishwasher is 20 min and the time to unpack each refrigerator is 10 min. The total time for unpacking is 70 min. This situation can be modelled by $20x + 10y \leq 70$, which can be simplified by dividing through by 10.

b.



The shaded area is the solution to the system of inequalities, but only the dots represent the feasible solutions. The solution must be a whole number because you cannot pack part of a dishwasher or refrigerator.

Activity 3 (continued)

7. a. The inequalities $x \geq 0$ and $y \geq 0$ are used as part of the system because the variables for the number of speakers must be non-negative.

The term $3x$ represents the time used to assemble System A, and the term $4y$ represents the time to assemble System B. Therefore, the expression $3x + 4y$ represents the total time used for assembling both systems.

The inequality $3x + 4y \leq 480$ is used as part of the system of inequalities because the total hours for speaker assembly must be less than or equal to 480 hours.

The inequality $2x + 6y \leq 540$ is used as a part of the system of inequalities because the total number of hours must be less than or equal to 540.

b. $3x + 4y \leq 480$

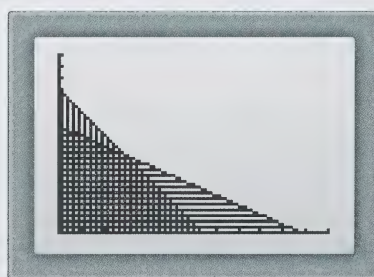
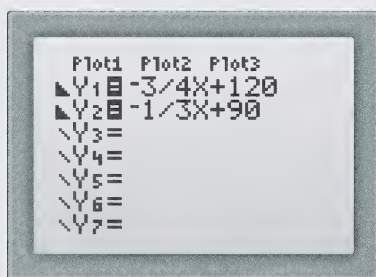
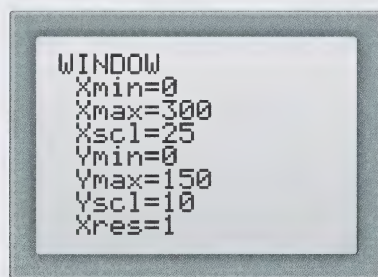
$$4y \leq -3x + 480$$

$$y \leq -\frac{3}{4}x + 120$$

$2x + 6y \leq 540$

$$6y \leq -2x + 540$$

$$y \leq -\frac{1}{3}x + 90$$



- c. Both x and y are discrete variables. They can only take on values that are whole numbers, since only whole numbers of speaker systems are of interest.

6. Textbook exercise “Communicating the Ideas,” p. 189

Answers may vary. A sample answer is given.

Graphing systems of linear inequalities is similar to graphing systems of linear equations. Both involve graphing lines, and both are solved by the concept of crossing graphs—either intersecting lines or overlapping half-planes. Both systems can have zero solutions. For systems of linear equations, this occurs when the lines involved are parallel. For systems of linear inequalities, this occurs when the boundary lines are parallel and the half-plane above the upper boundary line and the half-plane below the lower boundary line are shaded.

Graphing the two types of systems are also different. For systems of inequalities, the boundary lines may be solid or dotted. Also, for systems of linear inequalities, the solution set is due to the overlapping half-planes, not the intersection of lines—often resulting in the solution being a region of the coordinate plane and not just a single point or line.

Activity 4: Modelling a Problem Situation

1. a. $7x$ b. $x \leq 18$ c. $y \geq 5$ d. $x + y \leq 25$

2. Textbook exercises 2 and 4 of “Exercises: Checking Your Skills,” pp. 195 and 196

2. a. Let x represent the time (in hours) Larry plays basketball and y be the time (in hours) he plays golf.

b.

	Basketball	Golf	Total
Variable	x	y	
Time (h)	x	y	≤ 12
Calories/h	600	350	
Total Calories	$600x$	$350y$	≥ 5000

Time spent exercising cannot be negative.

$\therefore x \geq 0$ and $y \geq 0$

These are implicit constraints.

Time spent exercising per week is less than or equal to 12 h.

$\therefore x + y \leq 12$

Larry wishes to burn at least 5000 calories per week.

$\therefore 600x + 350y \geq 5000$

Activity 4 (continued)

Therefore, the system of inequalities for this problem is as follows:

$$\begin{array}{ll} x \geq 0 & \textcircled{1} \\ y \geq 0 & \textcircled{2} \\ x + y \leq 12 & \textcircled{3} \\ 600x + 350y \geq 5000 & \textcircled{4} \end{array}$$

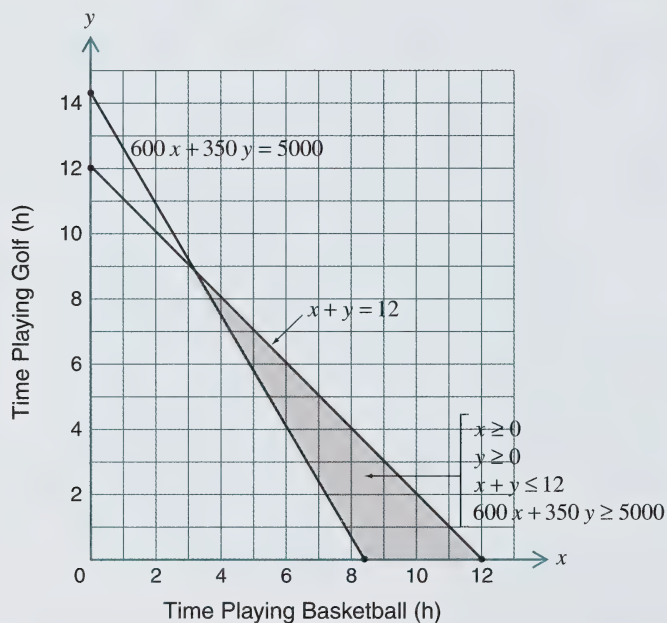
c. $x + y \leq 12$ $600x + 350y \geq 5000$

$$y \geq -x + 12$$

$$350y \geq -600x + 5000$$

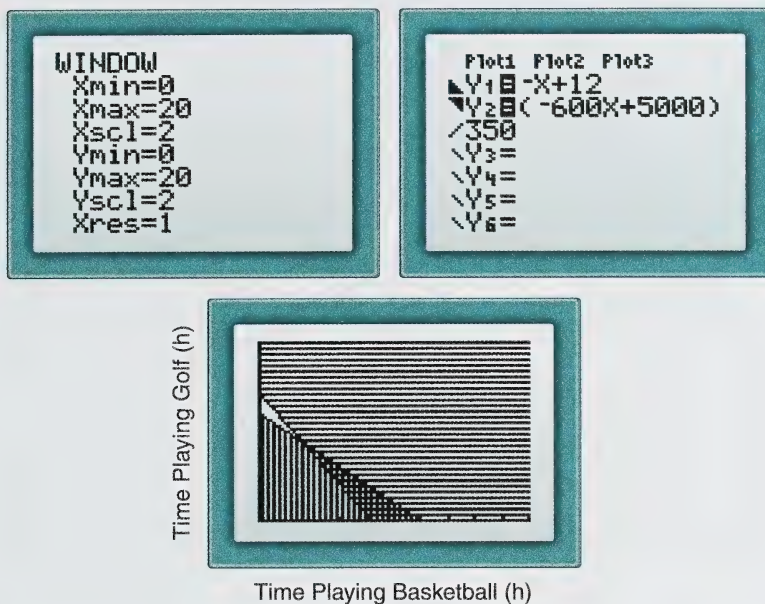
$$y \geq \frac{-600x + 5000}{350}$$

Method 1: Using Pencil and Paper



The time Larry could spend on each activity per week is given by any coordinate located in the shaded area.

Method 2: Using a Graphing Calculator



The time Larry could spend on each activity is given by any coordinate located in the double-shaded region.

4. a. Let x be the time (in hours) spent walking and y be the time (in hours) spent swimming.

b.

	Walking	Swimming	Total
Time (h)	x	y	≤ 5
Cost/h (\$)	0	4	
Total Cost (\$)	$0x$	$4y$	≤ 12
Calories/h	500	600	
Total Calories	$500x$	$600y$	≥ 2900

The system of inequalities for this problem is as follows:

$$\begin{array}{ll}
 x \geq 0 & \textcircled{1} \\
 y \geq 0 & \textcircled{2} \\
 x + y \leq 5 & \textcircled{3} \\
 4y \leq 12 & \textcircled{4} \\
 500x + 600y \geq 2900 & \textcircled{5}
 \end{array}
 \left. \begin{array}{l} \textcircled{1} \\ \textcircled{2} \end{array} \right\} \begin{array}{l} \text{These are implicit constraints. Time must be positive.} \\ \leftarrow \text{Holly can spend no more than 5 h per week.} \\ \leftarrow \text{Holly is willing to spend up to \$12.} \\ \leftarrow \text{Holly wants to burn at least 2900 calories.} \end{array}$$

Activity 4 (continued)

c. $x + y \leq 5$

$y \leq -x + 5$

$4y \leq 12$

$y \leq 3$

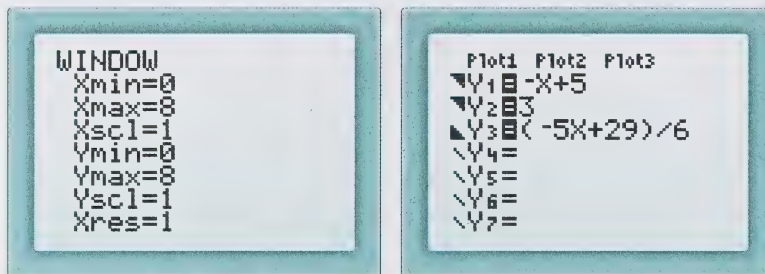
$500x + 600y \geq 2900$

$5x + 6y \geq 29$

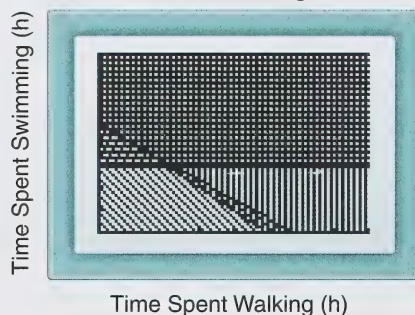
$6y \geq -5x + 29$

$y \geq \frac{-5x + 29}{6}$

Because there are more than two inequalities to graph, use reverse shading to find the solution.



reverse shading



The entire display is shaded. This shows that it is not possible to develop an exercise program that meets every constraint of the situation.

3. Textbook exercises 1 and 2 of “Discussing the Ideas,” p. 195

1. It is not stated that Marie must swim or play tennis only in full-hour increments.
2. Since only whole numbers of garments make sense in the problem context, only ordered pairs consisting of whole numbers within the unshaded region are possible solutions.

4. Textbook exercise 5 of “Exercises: Checking Your Skills,” p. 196

5. Let x be the number of mountain bikes and y be the number of touring bikes.

The storeowner wishes to buy at least 10 of each model.

$$\therefore x \geq 10 \text{ and } y \geq 10$$

The storeowner does not want to spend more than \$4400 in total.

$$\therefore 200x + 160y \leq 4400$$

$$5x + 4y \leq 110$$

Therefore, the system of inequalities for this problem is as follows:

$$x \geq 10 \quad (1)$$

$$y \geq 10 \quad (2)$$

$$5x + 4y \geq 110 \quad (3)$$

5. Textbook exercise 7 of “Exercises: Checking Your Skills,” p. 197

7. a. Let x be the length (in metres) and y be the width (in metres).

The system of inequalities for this problem is as follows:

$$x > 0 \quad (1)$$

$$y > 0 \quad (2)$$

$$y < x \quad (3)$$

$$xy \geq 280 \quad (4)$$

$$2(x + y) \leq 120$$

$$2x + 2y \leq 120$$

$$x + y \leq 60 \quad (5)$$

} These are implicit constraints. The dimensions must be positive to have any kind of pen.

← The length of the pen must be greater than the width.

← The area must be greater than 280 m^2 .

← The perimeter can be no more than 120 m.

Activity 4 (continued)

b. $y < x$

$xy \geq 280$

$x + y \leq 60$

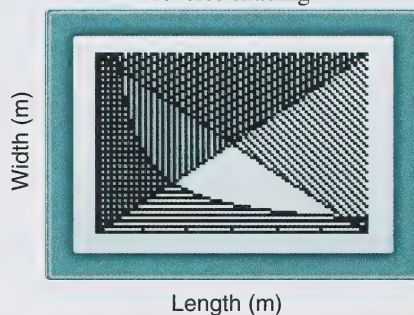
$y \geq \frac{280}{x}$

$y \leq -x + 60$

Because there are more than two inequalities to graph, use reverse shading.



reverse shading



6. You use inequalities to represent the constraints of the situation.
7. Implicit constraints are inferred from the problem situation. Implicit constraints are not stated directly.

Activity 5: Optimization Problems

1. Textbook exercises 2 to 10 of “Investigation: Maximizing Profits,” pp. 200 and 201

2. $P = 10x + 15y$

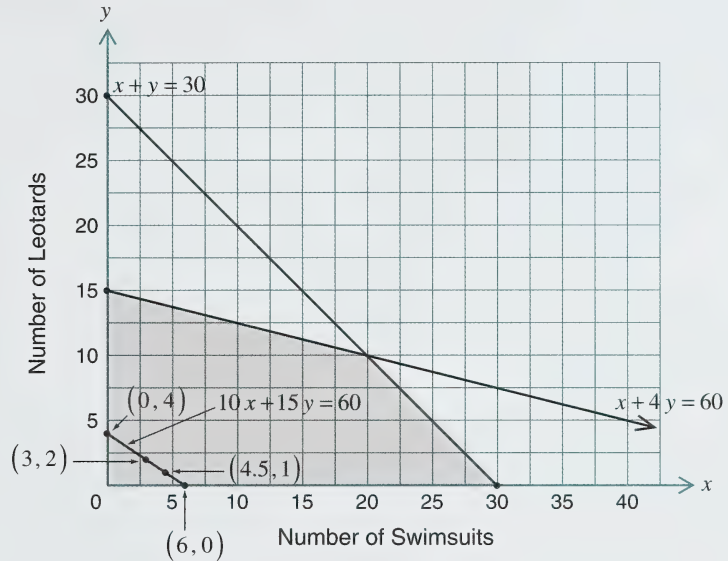
3. $P = 10x + 15y$

$$60 = 10x + 15y$$

$$15y = -10x + 60$$

$$y = \frac{-10x}{15} + \frac{60}{15}$$

$$y = -\frac{2}{3}x + 4$$



4. Refer to the graph in exercise 3 for the plotted points.

The point $(6, 0)$ means that 6 swimsuits and no leotards are made.

The point $(4.5, 1)$ means that 4.5 swimsuits and 1 leotard are made.

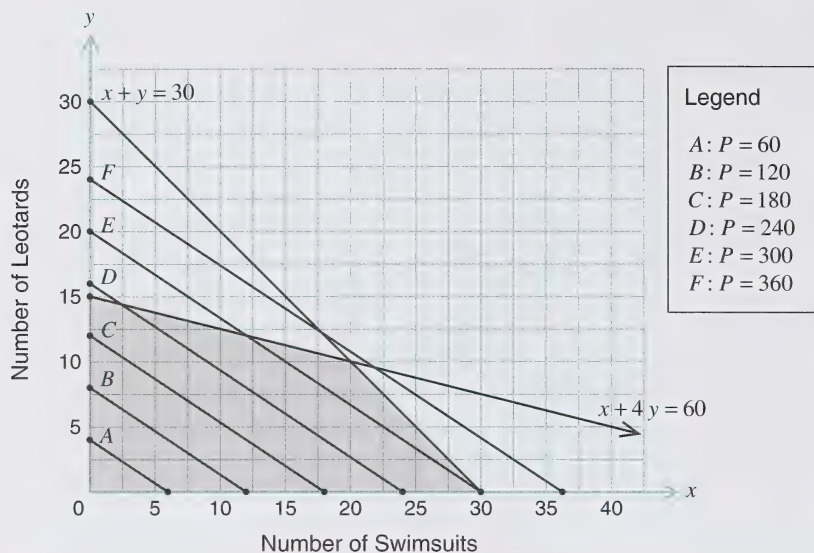
The point $(3, 2)$ means that 3 swimsuits and 2 leotards are made.

The point $(0, 4)$ means that no swimsuits and 4 leotards are made.

Only $(4.5, 1)$ is not feasible because you cannot sell part of a one-piece swimsuit.

Activity 5 (continued)

5.



Profit Level	Constraint Form	Interpretation
B $P = 120$	$(0, 8)$	no swimsuits and 8 leotards
	$(6, 4)$	6 swimsuits and 4 leotards
C $P = 180$	$(6, 8)$	6 swimsuits and 8 leotards
	$(12, 4)$	12 swimsuits and 4 leotards
D $P = 240$	$(12, 8)$	12 swimsuits and 8 leotards
	$(18, 4)$	18 swimsuits and 4 leotards
E $P = 300$	$(6, 16)$	6 swimsuits and 16 leotards
	$(18, 8)$	18 swimsuits and 8 leotards
F $P = 360$	$(12, 16)$	12 swimsuits and 16 leotards
	$(30, 4)$	30 swimsuits and 4 leotards

6. The profit lines are parallel.
7. The point $(20, 10)$ is closest to the $P = 360$ line. This point will have the largest profit. The point $(0, 0)$ will have the minimum profit; it is furthest away from the $P = 360$ line. (See graph in the answer to exercise 5 of this investigation.)

Check that $(0, 0)$ is the point within the solution region with the lowest profit.

$$\begin{aligned}
 P &= 10x + 15y \\
 &= 10(0) + 15(0) \\
 &= 0
 \end{aligned}$$

This is the lowest possible profit in this situation.

Check that $(20, 10)$ is the point within the solution region with highest profit.

$$\begin{aligned}
 P &= 10x + 15y \\
 &= 10(20) + 15(10) \\
 &= 200 + 150 \\
 &= 350
 \end{aligned}$$

The neighbouring feasible solutions are $(19, 9)$, $(19, 10)$, and $(20, 9)$.

$$\begin{aligned}
 \text{At } (19, 9), P &= 10x + 15y \\
 &= 10(19) + 15(9) \\
 &= 190 + 135 \\
 &= 325
 \end{aligned}$$

$$\begin{aligned}
 \text{At } (19, 10), P &= 10x + 15y \\
 &= 10(19) + 15(10) \\
 &= 190 + 150 \\
 &= 340
 \end{aligned}$$

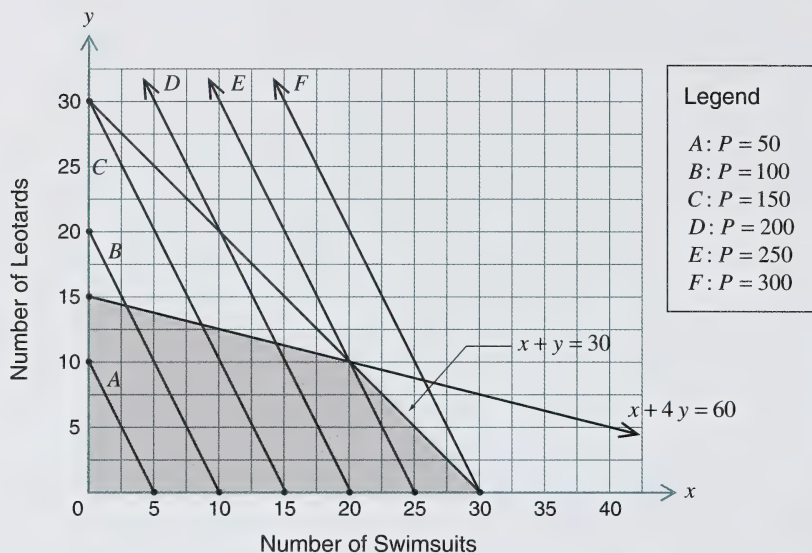
$$\begin{aligned}
 \text{At } (20, 9), P &= 10x + 15y \\
 &= 10(20) + 15(9) \\
 &= 200 + 135 \\
 &= 335
 \end{aligned}$$

It is very likely, based on these calculations, that $(20, 10)$ gives the highest profit. **Note:** A point like $(20, 20)$ will give more profit, but it is beyond the production of this manufacturer.

Activity 5 (continued)

8. Answers may vary depending on the profit values chosen.

The profit function is $P = 10x + 5y$. See the following graph for these profit lines.



The profit lines are all parallel.

Profit Level		Selected Points	Interpretation
A	$P = 50$	(2, 6)	2 swimsuits and 6 leotards
		(4, 2)	4 swimsuits and 2 leotards
B	$P = 100$	(4, 12)	4 swimsuits and 12 leotards
		(8, 4)	8 swimsuits and 4 leotards
C	$P = 150$	(6, 18)	6 swimsuits and 18 leotards
		(12, 6)	12 swimsuits and 6 leotards
D	$P = 200$	(12, 16)	12 swimsuits and 16 leotards
		(16, 8)	16 swimsuits and 8 leotards

E	$P = 250$	$(20, 10)$	20 swimsuits and 10 leotards
		$(22, 6)$	22 swimsuits and 6 leotards
F	$P = 300$	$(26, 8)$	26 swimsuits and 8 leotards
		$(30, 0)$	30 swimsuits and 0 leotards

The point $(30, 0)$ is the only point on the $P = 300$ line that lies within the solution region. Therefore, this point will have the largest profit.

The point $(0, 0)$ will have the minimum profit; it is furthest away from the $P = 300$ line.

Check that $(0, 0)$ is the point within the solution region with the lowest profit.

$$\begin{aligned}
 P &= 10x + 5y \\
 &= 10(0) + 5(0) \\
 &= 0
 \end{aligned}$$

This is the lowest possible profit in this situation.

Check that $(30, 0)$ is the point within the solution region with the highest profit.

$$\begin{aligned}
 P &= 10x + 5y \\
 &= 10(30) + 5(0) \\
 &= 300
 \end{aligned}$$

The neighbouring feasible solutions are $(29, 1)$ and $(28, 2)$.

$$\begin{aligned}
 \text{At } (29, 1), P &= 10x + 5y \\
 &= 10(29) + 5(1) \\
 &= 290 + 5 \\
 &= 295
 \end{aligned}$$

$$\begin{aligned}
 \text{At } (28, 2), P &= 10x + 5y \\
 &= 10(28) + 5(2) \\
 &= 280 + 10 \\
 &= 290
 \end{aligned}$$

It is very likely that $(30, 0)$ gives the highest profit.

Activity 5 (continued)

9. The minimum is still $(0, 0)$. The maximum, however, is now the lower-right vertex of the shaded region, $(30, 0)$.

The profit at this point is \$300.

10. The maximum and minimum points occur somewhere on the boundary of the solution region.

2. Textbook exercise 5 of “Exercises: Checking Your Skills,” p. 208

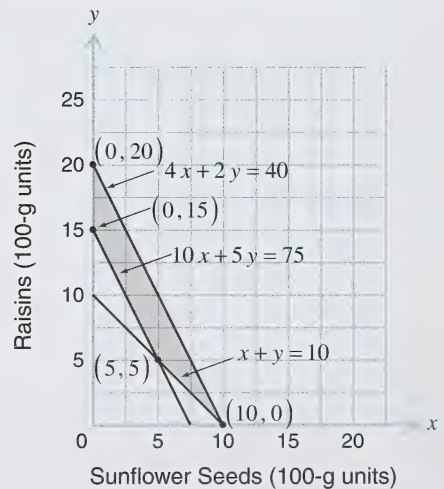
5. Let x be the number of units of sunflower seeds (in 100-g units) and y be the number of units of raisins (in 100-g units).

	Sunflower Seeds	Raisins	Total
Amount (100-g units)	x	y	≥ 10
Dietary Fibre (g)	$10x$	$5y$	≥ 75
Iron (mg)	$4x$	$2y$	≤ 40
Cost (\$)	$12x$	$10y$	Minimum

Develop the inequalities.

- | | |
|--------------------|-----|
| $x \geq 0$ | (1) |
| $y \geq 0$ | (2) |
| $x + y \geq 10$ | (3) |
| $10x + 5y \geq 75$ | (4) |
| $4x + 2y \leq 40$ | (5) |

The objective function is $C = 12x + 10y$.



$$\begin{aligned}\text{At } (0, 15), C &= 12x + 10y \\ &= 12(0) + 10(15) \\ &= 150\end{aligned}$$

$$\begin{aligned}\text{At } (0, 20), C &= 12x + 10y \\ &= 12(0) + 10(20) \\ &= 200\end{aligned}$$

$$\begin{aligned}\text{At } (5, 5), C &= 12x + 10y \\ &= 12(5) + 10(5) \\ &= 60 + 50 \\ &= 110\end{aligned}$$

$$\begin{aligned}\text{At } (10, 0), C &= 12x + 10y \\ &= 12(10) + 10(0) \\ &= 120\end{aligned}$$

The minimum cost subject to the constraints occur at $(5, 5)$. Therefore, 5 units (500 g) of sunflower seeds and 5 units (500 g) of raisins should be used to minimize the cost of producing this mixture.

3. Textbook exercises 1, 2, 3, 4, and 7 of “Exercises: Checking Your Skills,” pp. 207 and 208

1. a. Calculate the value of the objective function, $C = 3x + 5y$, at each vertex.

$$\begin{aligned}\text{At } (0, 8), C &= 3x + 5y \\ &= 3(0) + 5(8) \\ &= 40\end{aligned}$$

$$\begin{aligned}\text{At } (3, 3), C &= 3x + 5y \\ &= 3(3) + 5(3) \\ &= 9 + 15 \\ &= 24\end{aligned}$$

$$\begin{aligned}\text{At } (8, 0), C &= 3x + 5y \\ &= 3(8) + 5(0) \\ &= 24\end{aligned}$$

The optimal value is 40, which occurs at $(0, 8)$.

- b. Calculate the value of the objective function, $T = x + 4y$, at each vertex.

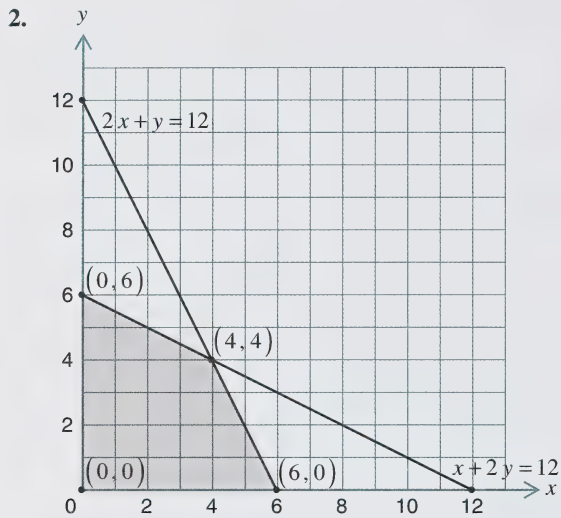
$$\begin{aligned}\text{At } (0, 3), T &= x + 4y \\ &= 0 + 4(3) \\ &= 12\end{aligned}$$

$$\begin{aligned}\text{At } (2, 4), T &= x + 4y \\ &= 2 + 4(4) \\ &= 2 + 16 \\ &= 18\end{aligned}$$

$$\begin{aligned}\text{At } (5, 0), T &= x + 4y \\ &= 5 + 4(0) \\ &= 5\end{aligned}$$

The optimal value is 18, which occurs at $(2, 4)$.

Activity 5 (continued)



Calculate the values of the objective function, $T = 5x + 7y$, at each vertex.

$$\begin{aligned}\text{At } (0, 0), T &= 5x + 7y \\ &= 5(0) + 7(0) \\ &= 0 + 0 \\ &= 0\end{aligned}$$

$$\begin{aligned}\text{At } (0, 6), T &= 5x + 7y \\ &= 5(0) + 7(6) \\ &= 0 + 42 \\ &= 42\end{aligned}$$

$$\begin{aligned}\text{At } (4, 4), T &= 5x + 7y \\ &= 5(4) + 7(4) \\ &= 20 + 28 \\ &= 48\end{aligned}$$

$$\begin{aligned}\text{At } (6, 0), T &= 5x + 7y \\ &= 5(6) + 7(0) \\ &= 30 + 0 \\ &= 30\end{aligned}$$

The optimal values are 0 (minimum) and 48 (maximum).

3. Let x be the number of jackets of the first type and y be the number of jackets of the second type.

	First Type	Second Type	Total
Number of Jackets	x	y	
Cutting Time (min)	$20x$	$10y$	≤ 720
Sewing Time (min)	$10x$	$15y$	≤ 480
Profit (\$)	$20x$	$15y$	Maximum

$$12 \text{ h} = 12 \times 60 \\ = 720 \text{ min}$$

$$8 \text{ h} = 8 \times 60 \\ = 480 \text{ min}$$

Constraints:

$$x \geq 0$$

(1)

$$y \geq 0$$

(2)

$$20x + 10y \leq 720$$

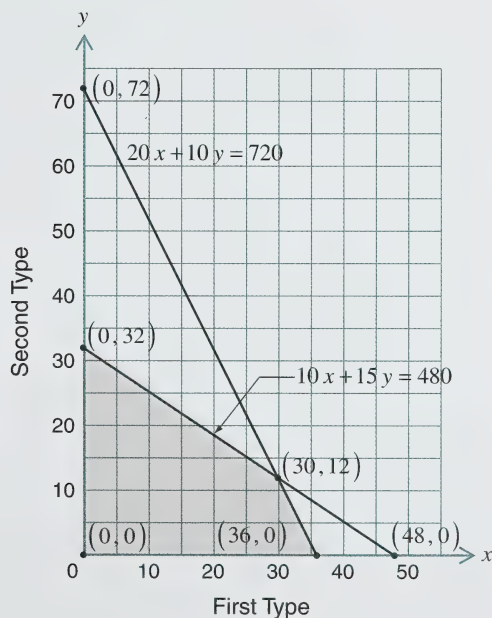
(3)

$$10x + 15y \leq 480$$

(4)

Remember: If you are using a graphing calculator, you will have to transform these inequalities into slope-intercept form, $y = mx + b$.

Graph the system of inequalities, and find the vertices of the solution region.



Activity 5 (continued)

Calculate the value of the objective function, $P = 20x + 15y$, at each vertex.

$$\begin{aligned}\text{At } (0, 0), P &= 20x + 15y \\ &= 20(0) + 15(0) \\ &= 0\end{aligned}$$

$$\begin{aligned}\text{At } (0, 32), P &= 20x + 15y \\ &= 20(0) + 15(32) \\ &= 480\end{aligned}$$

$$\begin{aligned}\text{At } (30, 12), P &= 20x + 15y \\ &= 20(30) + 15(12) \\ &= 600 + 180 \\ &= 780\end{aligned}$$

$$\begin{aligned}\text{At } (36, 0), P &= 20x + 15y \\ &= 20(36) + 15(0) \\ &= 720\end{aligned}$$

To maximize profits, the company should produce 30 jackets of the first type and 12 of the second type.

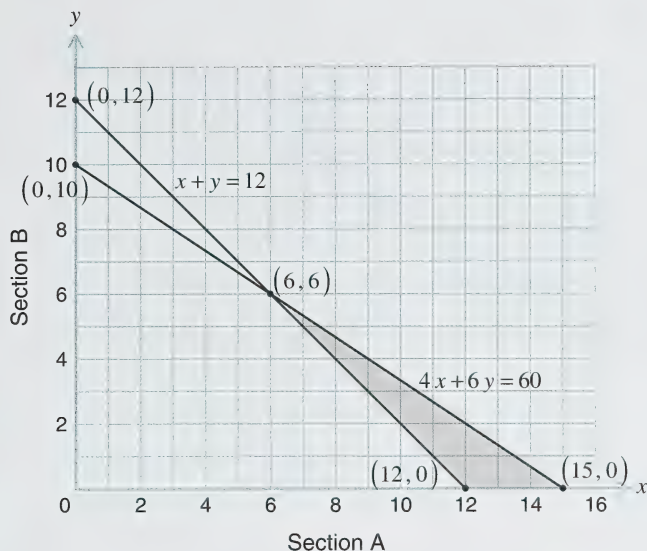
4.

	Section A	Section B	Total
Number of Questions	x	y	≥ 12
Time to Solve (min)	$4x$	$6y$	≤ 60
Score	$6x$	$10y$	Maximum

Constraints:

$$\begin{array}{ll}x \geq 0 & \textcircled{1} \\ y \geq 0 & \textcircled{2} \\ x + y \geq 12 & \textcircled{3} \\ 4x + 6y \leq 60 & \textcircled{4}\end{array}$$

Graph the system of inequalities, and find the vertices of the solution region.



Calculate the value of the objective function, $M = 6x + 10y$, at each vertex.

$$\begin{aligned}\text{At } (6, 6), M &= 6x + 10y \\ &= 6(6) + 10(6) \\ &= 36 + 60 \\ &= 96\end{aligned}$$

$$\begin{aligned}\text{At } (12, 0), M &= 6x + 10y \\ &= 6(12) + 10(0) \\ &= 72 + 0 \\ &= 72\end{aligned}$$

$$\begin{aligned}\text{At } (15, 0), M &= 6x + 10y \\ &= 6(15) + 10(0) \\ &= 90 + 0 \\ &= 90\end{aligned}$$

To obtain the maximum score, you should answer 6 questions from each section.

7. a.

	First Type	Second Type	Total
Number of Hats	x	y	
Cutting Time (min)	$6x$	$2y$	≤ 75
Sewing Time (min)	$1x$	$4y$	≤ 40
Profit (\$)	$8x$	$5y$	Maximum

Activity 5 (continued)

Constraints:

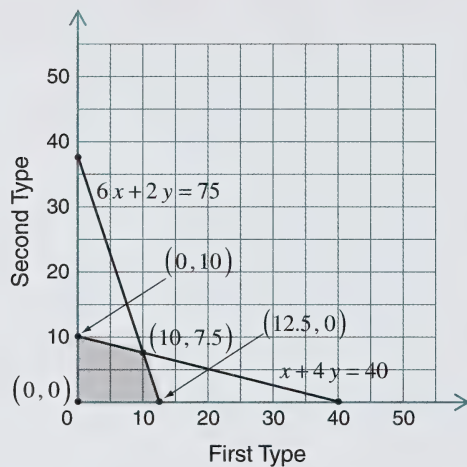
$$x \geq 0 \quad (1)$$

$$y \geq 0 \quad (2)$$

$$6x + 2y \leq 75 \quad (3)$$

$$x + 4y \leq 40 \quad (4)$$

Graph the system of inequalities, and find the vertices of the solution region.



Calculate the value of the objective function, $P = 8x + 5y$, at each vertex.

$$\begin{aligned} \text{At } (0, 0), P &= 8x + 5y \\ &= 8(0) + 5(0) \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{At } (0, 10), P &= 8x + 5y \\ &= 8(0) + 5(10) \\ &= 50 \end{aligned}$$

$$\begin{aligned} \text{At } (10, 7.5), P &= 8x + 5y \\ &= 8(10) + 5(7.5) \\ &= 80 + 37.5 \\ &= 117.50 \end{aligned}$$

$$\begin{aligned} \text{At } (12.5, 0), P &= 8x + 5y \\ &= 8(12.5) + 5(0) \\ &= 100 \end{aligned}$$

According to the simple interpretation of linear programming, the company should produce 10 hats of the first type and 7.5 hats of the second type.

- b. Assuming the factory will continue making hats after this 2-h period, the result is probably reasonable. Otherwise, the result in exercise 7.a. is not reasonable. There is no profit to be made from half of a hat. Only whole numbers of hats are marketable.

Therefore, compare the profits of nearby points within the solution region but closest to the boundary.

$$\text{At } (9, 7), P = 8x + 5y$$

$$= 8(9) + 5(7)$$

$$= 72 + 35$$

$$= 107$$

$$\text{At } (10, 7), P = 8x + 5y$$

$$= 8(10) + 5(7)$$

$$= 80 + 35$$

$$= 115$$

To maximize profits, the company should make 10 hats of the first type and 7 hats of the second type in this 2-h time slot.

4. Textbook exercise 1 of “Discussing the Ideas,” p. 206

1. A solution is any point that satisfies the system of inequalities. Such a point lies within the solution region. The optimal solution is the value of the objective function that is either a maximum or a minimum, depending on the problem situation.

5. Textbook exercise “Communicating the Ideas,” p. 209

The vertices are important because the optimal value of the objective function occurs on (at least) one of the vertices.

A step-by-step procedure to determine the coordinates of the vertices of a solution region of an optimization problem is as follows:

Step 1: Assign variables to the values in the problem situation.

Step 2: Set up a system of inequalities according to the information in the problem.

Step 3: Graph the system of inequalities on a coordinate grid or using a graphing calculator.

Step 4: Substitute the coordinates of each vertex of the solution region into the objective function to find the optimal value(s). **Note:** If using a graphing calculator, use the features from the CALCULATE menu to determine each vertex.

Follow-up Activities

Textbook exercise 9 of Part B of “What Should I Be Able to Do?,” p. 215

9. Let x be the number of first-class passengers and y be the number of economy-class passengers.

	First Class	Economy	Total
Number of Passengers	x	y	≤ 48
Baggage (kg)	$60x$	$20y$	≤ 1440
Revenue	$1000x$	$500y$	Maximum

Constraints:

$$x \geq 0 \quad (1)$$

$$x + y \leq 48 \quad (3)$$

$$y \geq 0 \quad (2)$$

$$60x + 20y \leq 1440 \quad (4)$$

Use your graphing calculator to graph the inequalities. Convert the inequalities to slope-intercept form, $y \leq mx + b$, where $\blacksquare$ represents either of the inequality signs.

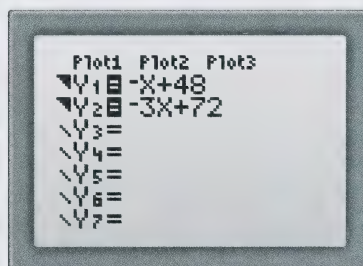
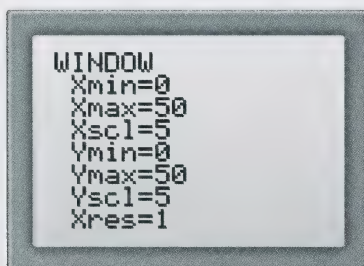
$$x + y \leq 48$$

$$60x + 20y \leq 1440$$

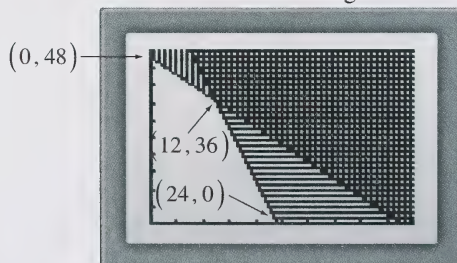
$$y \leq -x + 48$$

$$3x + y \leq 72$$

$$y \leq -3x + 72$$

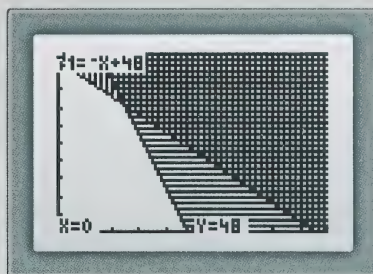


reverse shading



To find the vertex $(0, 48)$, use the Value feature from the CALCULATE menu on the graph of Y_1 .

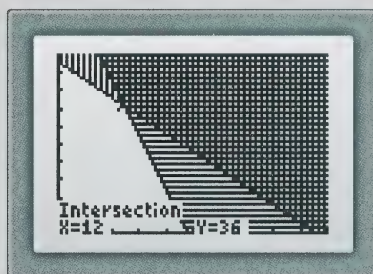
2nd **[CALC]** **1** (1:value) **0** **ENTER**



To find the vertex $(12, 36)$, use the Intersect feature from the CALCULATE menu.

2nd **[CALC]** **5** (5:intersect) **ENTER**

ENTER **ENTER**



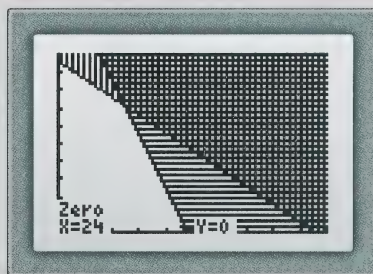
To find the vertex $(24, 0)$, use the Zero feature from the CALCULATE menu on the graph of Y_2 .

Step 1: Choose the graph of Y_2 .

2nd **[CALC]** **2** (2:zero) **▼**

Step 2: Choose the upper and lower bounds.

Step 3: Find the zero.



Follow-up Activities (continued)

Calculate the value of the objective function, $R = 1000x + 500y$, at each vertex.

$$\begin{aligned}\text{At } (0, 48), R &= 1000x + 500y \\ &= 1000(0) + 500(48) \\ &= 0 + 24\,000 \\ &= 24\,000\end{aligned}$$

$$\begin{aligned}\text{At } (12, 36), R &= 1000x + 500y \\ &= 1000(12) + 500(36) \\ &= 12\,000 + 18\,000 \\ &= 30\,000\end{aligned}$$

$$\begin{aligned}\text{At } (24, 0), R &= 1000x + 500y \\ &= 1000(24) + 500(0) \\ &= 24\,000 + 0 \\ &= 24\,000\end{aligned}$$

The maximum revenue occurs at $(12, 36)$. Therefore, 12 first-class passengers and 36 economy-class passengers give a maximum revenue.

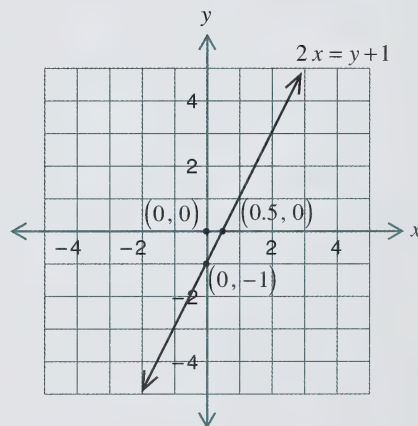
Extra Help

1. **Step 1:** Graph the corresponding equation, $2x = y + 1$.

Step 2: Apply a test point, such as $(0, 0)$.

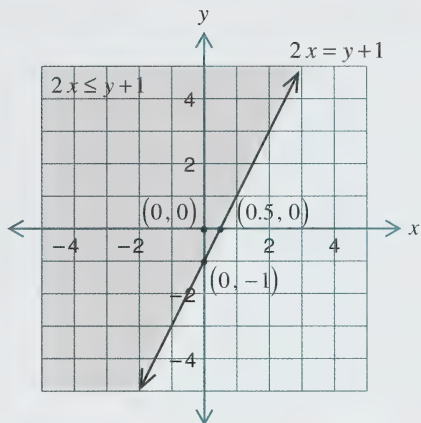
LS	RS
$2x$	$y + 1$
$= 2(0)$	$= (0) + 1$
$= 0$	$= 1$
LS	$\leq$ RS

The test point satisfies the inequality.



Step 3: Shade the appropriate half-plane.

Because the test point satisfies the inequality and is above the boundary line, the upper half-plane should be shaded.



2. **Step 1:** Graph the corresponding equation, $3x - y = 0$, as a broken line because the $>$ sign is used.

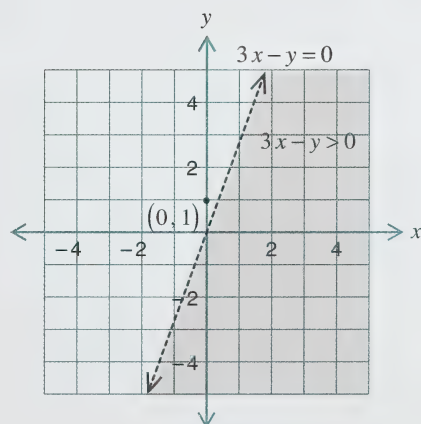
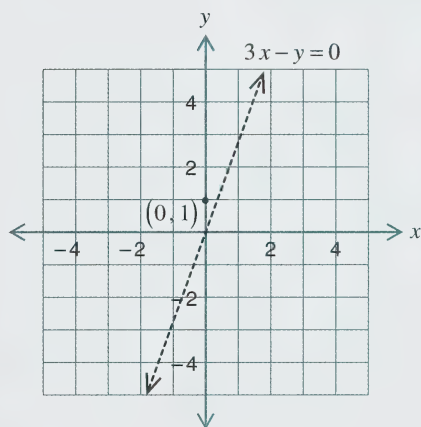
Step 2: Apply a test point, such as $(0, 1)$. **Note:** The point $(0, 0)$ is not a suitable test point because it is on the boundary line.

LS	RS
$3x - y$	0
$= 3(0) - (1)$	
$= -1$	
LS	$\nless$ RS

The test point does not satisfy the inequality.

Step 3: Shade the appropriate half-plane.

Since the test point does not satisfy the inequality, shade the half-plane on the opposite side of the boundary line from the test point. In this case, the lower half-plane should be shaded.



Follow-up Activities (continued)

Enrichment

1. The maximum point is the centre of the concentric circles in the 2-D graph. In the 3-D graph the maximum point is the highest point of the rounded surface.
2. The maximum of a linear function occurs at one of the vertices of a solution region. The maximum of a non-linear function can occur anywhere in the solution region, not necessarily at the vertices.

Module Project: Trail Mix

Completing the Project

1. Textbook exercise 1 of “Finding Feasible Snack Mixes,” p. 198

1. Answers will vary because dietary recommendations vary.

Any recommendation should be based on sound dietary information, such as the following:

- Dietary fibre is good for you. You should have approximately 30 g per day.
- Protein provides the building (and repair) materials for the body as well as energy.
- Carbohydrates are important as a source of energy.
- Fats are highly concentrated sources of energy. Fat intake should be no more than 30% of your daily food intake in terms of calories or no more than approximately 75 g in absolute terms of mass.

A good recommendation is to increase the non-fat components of the trail mix so the relative fat content is reduced. The present mix is too high in fat content. As a snack food, it would comprise a small part of your diet but half of the recommended fat intake.

2. Proposed mixture: 400 g of semisweet chocolate
800 g of seedless raisins

Clearly these meet constraints of the first two bullets. Check that the fat is reduced by 25%.

A 25% fat-reduced mixture has $33.2 \text{ g} \times 0.75 = 24.9 \text{ g}$ of fat per 100 g.

$$\begin{aligned}\text{Total fat in mixture} &= \left(400 \times \frac{30}{100}\right) + \left(800 \times \frac{0.46}{100}\right) \\ &= 120 + 3.68 \\ &= 123.68 \text{ g of fat}\end{aligned}$$

The relative fat content of the mixture per 100 g is

$$\begin{aligned}\frac{123.68}{400+800} &= \frac{123.68}{1200} \\ &= \frac{123.68 \div 12}{1200 \div 12} \\ &= \frac{10.31 \text{ g fat}}{100 \text{ g}}\end{aligned}$$

There are 10.31 g of fat per 100 g in this mixture. This is well below the fat reduction required in the third bullet.

Check that the trail mix is 5% higher in carbohydrates.

This requires the carbohydrate content to be $29.6 \text{ g} + (29.6 \text{ g} \times 0.05) = 31.08 \text{ g}$ carbohydrates per 100 g.

The total carbohydrates in the proposed mixture is

$$\begin{aligned}\text{Total carbohydrates} &= \left(400 \times \frac{63.1}{100}\right) + \left(800 \times \frac{79.13}{100}\right) \\ &= (252.4) + (633.0) \\ &= 885.4 \text{ g of carbohydrates}\end{aligned}$$

$$\begin{aligned}\text{Relative carbohydrate content} &= \frac{885.4}{400+800} \\ &= \frac{885.4 \div 12}{1200 \div 12} \\ &= \frac{73.78 \text{ g carbohydrates}}{100 \text{ g}}\end{aligned}$$

There are 73.78 g of carbohydrates per 100 g in this mixture. This is well above the 5% increase in carbohydrates for the fourth bullet.

Neither the dietary fibre nor the protein per 100 g was increased in the proposed mixture.

No adjustment in amounts is necessary.

Therefore, the mixture composed of 400 g of semisweet chocolate and 800 g of seedless raisins satisfies the four bulleted conditions.

Module Project (continued)

3. a. Let x represent one ingredient and y represent the other ingredient.

The trail mix has at least 1000 g in total.

$$\therefore x + y \geq 1000$$

At least 200 g of each ingredient must be used.

$$\therefore x \geq 200 \text{ and } y \geq 200$$

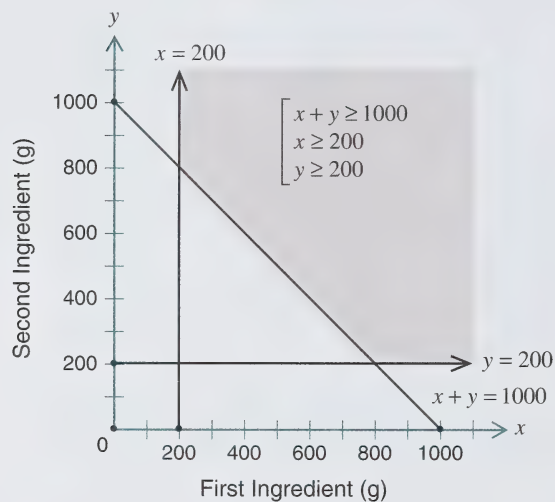
Therefore, the system of inequalities is as follows:

$$x + y \geq 1000 \quad (1)$$

$$x \geq 200 \quad (2)$$

$$y \geq 200 \quad (3)$$

b.



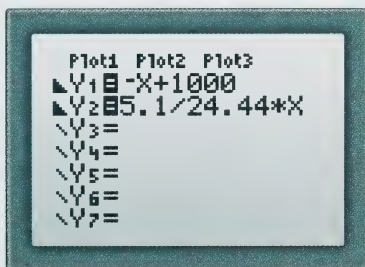
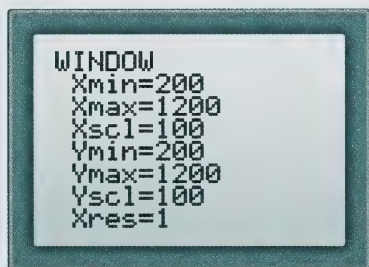
4. a. A mixture containing 25% less fat than the present trail mix contains the following amount of fat per 100 g of mix.

$$\begin{aligned}\text{Fat} &= 33.2 - 0.25(33.2) \\ &= 33.2 - 8.3 \\ &= 24.9 \text{ g/100 g of mix}\end{aligned}$$

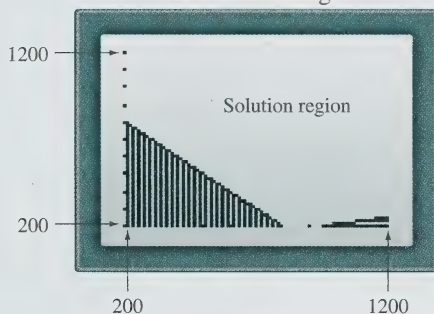
Therefore, the inequality that represents the third bullet is

$$\begin{aligned}\frac{30}{100}x + \frac{0.46}{100}y &\leq \frac{24.9}{100}(x+y) \\ 30x + 0.46y &\leq 24.9(x+y) \\ 30x + 0.46y &\leq 24.9x + 24.9y \\ 30x - 24.9x &\leq 24.9y - 0.46y \\ 5.1x &\leq 24.44y \\ 24.44y &\geq 5.1x \\ y &\geq \frac{5.1}{24.44}x\end{aligned}$$

b.



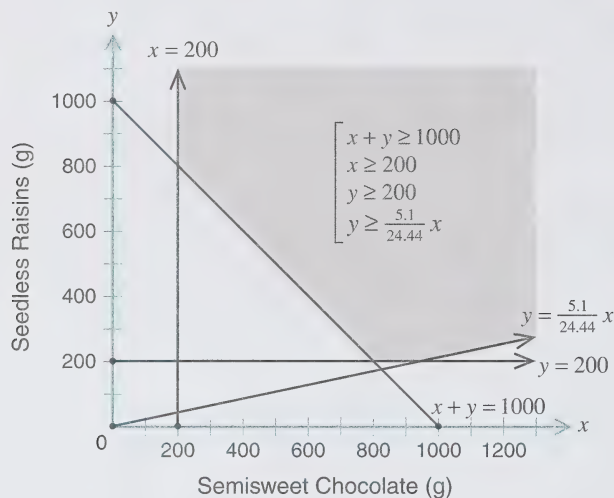
reverse shading



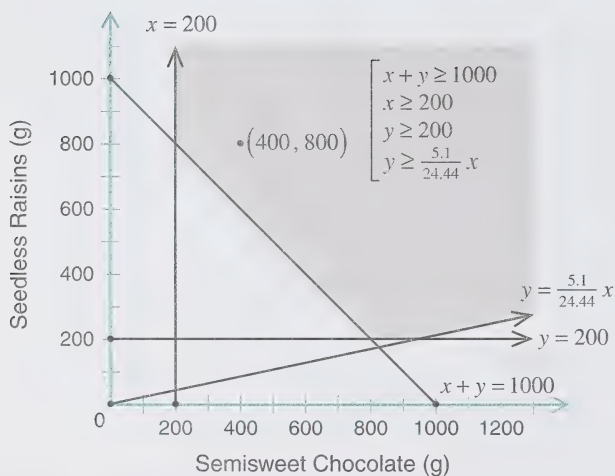
Notice that $X \min = 200$ and $Y \min = 200$ account for the constraints $x \geq 200$ and $y \geq 200$.

Module Project (continued)

Your sketch should now look like the following.



c.



d. Answers will vary.

You should check that your solution works. From exercise 4.b., the relative carbohydrate content (in grams per 100 g) of the mixture is

$$\text{Carbohydrate content} = \frac{\frac{63.1}{100}x + \frac{79.13}{100}y}{\frac{(x+y)}{100}}$$

The following table shows relative carbohydrate content (in grams per 100 g) for some possible mixtures. These values are obtained using the preceding formula.

		Seedless Raisins			
		200 g	500 g	800 g	1000 g
Semisweet Chocolate	200 g			75.9	76.5
	500 g		71.1	73.0	73.8
	800 g	66.3	69.3	71.1	72.0
	1000 g		68.4	70.2	71.1

None of these mixtures have adequate fibre or protein content. However, their ample carbohydrate content is enough to meet the constraint of the fourth bullet.

5. Three proposed mixtures are $(400, 800)$, $(1000, 500)$, and $(200, 800)$. That is,

- 400 g of semisweet chocolate and 800 g of seedless raisins
- 1000 g of semisweet chocolate and 500 g of seedless raisins
- 200 g of semisweet chocolate and 800 g of seedless raisins

$$\begin{aligned}
 \text{At } (400, 800), C &= 21\left(\frac{x}{100}\right) + 10\left(\frac{y}{100}\right) \\
 &= 21\left(\frac{400}{100}\right) + 10\left(\frac{800}{100}\right) \\
 &= 21(4) + 10(8) \\
 &= 84 + 80 \\
 &= 164\text{¢ or } \$1.64
 \end{aligned}$$

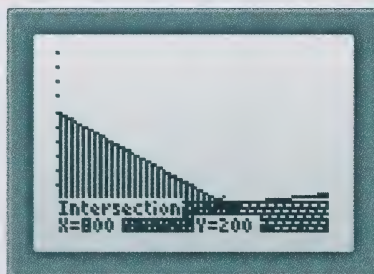
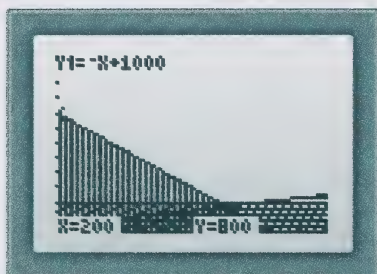
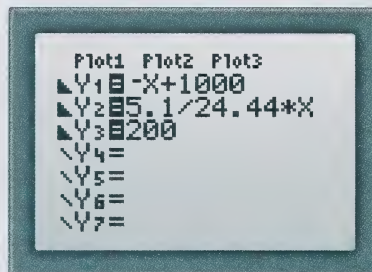
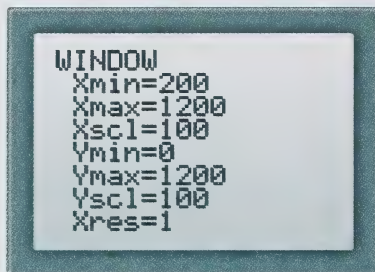
$$\begin{aligned}
 \text{At } (1000, 500), C &= 21\left(\frac{x}{100}\right) + 10\left(\frac{y}{100}\right) \\
 &= 21\left(\frac{1000}{100}\right) + 10\left(\frac{500}{100}\right) \\
 &= 21(10) + 10(5) \\
 &= 210 + 50 \\
 &= 260\text{¢ or } \$2.60
 \end{aligned}$$

$$\begin{aligned}
 \text{At } (200, 800), C &= 21\left(\frac{x}{100}\right) + 10\left(\frac{y}{100}\right) \\
 &= 21\left(\frac{200}{100}\right) + 10\left(\frac{800}{100}\right) \\
 &= 21(2) + 10(8) \\
 &= 42 + 80 \\
 &= 122\text{¢ or } \$1.22
 \end{aligned}$$

Module Project (continued)

6. a. The minimum cost occurs on the vertices of the solution region. From the graph in the answer to exercise 4.b., there are three vertices on the boundary of the region.

Use your graphing calculator to find the three vertices of the solution region where the inequalities intersect.



Now, check the cost at the vertices. The cost increases as you move a chosen point away from the line running through vertices $(200, 800)$ and $(800, 200)$. The third vertex is off the line; so, its associated cost will be higher and does not have to be checked.

$$\begin{aligned}
 \text{At } (200, 800), C &= 21\left(\frac{x}{100}\right) + 10\left(\frac{y}{100}\right) \\
 &= 21\left(\frac{200}{100}\right) + 10\left(\frac{800}{100}\right) \\
 &= 122¢ \text{ or } \$1.22
 \end{aligned}$$

$$\begin{aligned}
 \text{At } (800, 200), C &= 21\left(\frac{x}{100}\right) + 10\left(\frac{y}{100}\right) \\
 &= 21\left(\frac{800}{100}\right) + 10\left(\frac{200}{100}\right) \\
 &= 168 + 20 \\
 &= 188¢ \text{ or } \$1.88
 \end{aligned}$$

The least-expensive mixture is 200 g of semisweet chocolate and 800 g of seedless raisins. It costs \$1.22.

$$\begin{aligned}
 \text{b. Protein} &= \frac{\frac{4.2}{100}x + \frac{3.2}{100}y}{\frac{x+y}{100}} \\
 &= \frac{\frac{4.2}{100} \left(\frac{2}{200} \right) + \frac{3.2}{100} \left(\frac{8}{800} \right)}{\frac{800+200}{100}} \\
 &= \frac{8.4 + 25.6}{1000} \\
 &= \frac{34.0}{100} \\
 &= 3.4 \text{ g/100 g}
 \end{aligned}$$

$$\begin{aligned}
 \text{Total fat} &= \frac{\frac{30}{100}x + \frac{0.46}{100}y}{\frac{1000}{100}} \\
 &= \frac{\frac{30}{100} \left(\frac{2}{200} \right) + \frac{0.46}{100} \left(\frac{8}{800} \right)}{10} \\
 &= \frac{60 + 3.68}{10} \\
 &= \frac{63.68}{10} \\
 &\doteq 6.4 \text{ g/100 g}
 \end{aligned}$$

$$\begin{aligned}
 \text{Carbohydrate} &= \frac{\frac{63.1}{100}x + \frac{79.13}{100}y}{\frac{1000}{100}} \\
 &= \frac{\frac{63.1}{100} \left(\frac{2}{200} \right) + \frac{79.13}{100} \left(\frac{8}{800} \right)}{10} \\
 &= \frac{126.2 + 633.04}{10} \\
 &= \frac{759.24}{10} \\
 &\doteq 75.9 \text{ g/100 g}
 \end{aligned}$$

$$\begin{aligned}
 \text{Dietary fibre} &= \frac{\frac{5.9}{100}x + \frac{4.0}{100}y}{\frac{1000}{100}} \\
 &= \frac{\frac{5.9}{100} \left(\frac{2}{200} \right) + \frac{4.0}{100} \left(\frac{8}{800} \right)}{10} \\
 &= \frac{11.8 + 32.0}{10} \\
 &= \frac{43.8}{10} \\
 &\doteq 4.4 \text{ g/100 g}
 \end{aligned}$$

The protein, total fat, and dietary fibre of the least-expensive trail mix is considerably less than those in the company's mix. The carbohydrates, however, are considerably higher than the company's mixture.

7. Textbook exercises 10 and 11 of Part C of "What Should I Be Able to Do?," pp. 216 and 217

10. The mixes look reasonable.

The fat reduction exceeds the 25% reduction requirement for each mixture, and the carbohydrate content increased by more than 5% for each mixture. Thus, the criteria have been met.

Note that when the analysis was transcribed for a good copy, the student introduced a copying error.

Mixture #2 consists of 500 g chocolate and 500 g raisins (not 1500 g raisins).

11. The analysis is correct. However, an entry for Mixture #3, average dietary fibre content for 100 g, should have been entered as 7.55 not 57.55.

8. Answers will vary. The student felt that the reduced fat content (or overall nutritional value) is the most important factor in choosing the best trail mix.

IMAGE CREDITS

Welcome Page

PhotoDisc, Inc.

Page

7	collage: Image Club/StudioGear/EyeWire, Inc. (top and right), PhotoDisc, Inc. (bottom left)
9	PhotoDisc, Inc.
12–13	collage: PhotoDisc, Inc.
14	© 2000–2001www.arttoday.com
15	© 2000–2001www.arttoday.com
16	Image Club/StudioGear/EyeWire, Inc.
18	Image Club/StudioGear/EyeWire, Inc.
20	top: Image Club/StudioGear/EyeWire, Inc.
21	Corel Corporation
23	top: PhotoDisc, Inc. bottom: PhotoDisc, Inc.
24	left: PhotoDisc, Inc. right: Image Club/StudioGear/EyeWire, Inc.
26	© 2000–2001www.arttoday.com
27	top: Image Club/StudioGear/EyeWire, Inc. bottom: PhotoDisc, Inc.
29	© 2000–2001www.arttoday.com
30	middle: Image Club/StudioGear/EyeWire, Inc.
31	top: Image Club/StudioGear/EyeWire, Inc.
32	PhotoDisc, Inc.
35	middle: PhotoDisc, Inc.
36	PhotoDisc, Inc.
37	Image Club/StudioGear/EyeWire, Inc.
43	© 2000–2001www.arttoday.com
46	PhotoDisc, Inc.
47	PhotoDisc, Inc.

